Lehrstuhl für Automatentheorie
Institut für Theoretische Informatik, TU Dresden
Prof. Dr. F. Baader

Nöthnitzer Str. 46
01187 Dresden
Tel.: 0351/463–39167

7. Exercises for the Course 'Description Logics'

Exercise 25:

We consider another form of blocking, where an individual can be blocked by an individual that is not necessarily an ancestor: anywhere blocking. Instead of the depth of an individual and the ancestor relation, it uses the age of an individual and the relation $<$.
The *age* of an individual x ($age(x)$) is defined as 0 for old individuals and n for a new individual x, if x was generated by the nth application of the $\exists$-rule.
Let $\mathcal{A}$ be an ABox obtained by applying the tableau rules and the GCI rule to an initial ABox $\mathcal{A}_0$. A new individual x is *anywhere blocked* by an individual a in an ABox $\mathcal{A}$, iff

- $\{C \mid C(x) \in \mathcal{A}\} \subseteq \{D \mid D(a) \in \mathcal{A}\}$, and
- $age(a) < age(x)$.

Prove the following for this form of blocking:

a) soundness

b) completeness
Hint: For what subset of the complete tableau do we need to construct a model?

c) termination

Exercise 26:

Let $\mathcal{K} = (\mathcal{A}_0, \mathcal{T})$ be an $\mathcal{ALC}$-knowledge base, with $\mathcal{T}$ a general TBox. The *precompletion* of $\mathcal{K}$ is the set of ABoxes $\mathcal{M}$ that is produced by the tableau algorithm when starting with the set of ABoxes $\{\mathcal{A}_0\}$ and exhaustingly applying all rules except the $\exists$-rule. Do the following:

(a) Show that $\mathcal{K}$ is consistent iff there is an open $\mathcal{A} \in \mathcal{M}$ such that for all individual names a occurring in $\mathcal{A}$, the concept $C^a_{\mathcal{A}} := \bigsqcap_{C(a)\in\mathcal{A}} C$ is satisfiable w.r.t. $\mathcal{T}$.

Hint: For the "if" direction, proceed as follows. The correctness of the tableau algorithm for $\mathcal{ALC}$ implies that, if $C^a_{\mathcal{A}}$ is satisfiable, then exhaustively applying (all!) rules to the set of ABoxes $\{\{C^a_{\mathcal{A}}(a)\}\}$ yields a set $\mathcal{M}'$ that contains an open and complete ABox. Show how to join all these ABoxes to obtain an open and complete tableau for $\mathcal{A}$ and conclude that $\mathcal{A}_0$ is consistent w.r.t. $\mathcal{T}$.

(b) Use the result from (a) to prove that ABox consistency in $\mathcal{ALC}$ can be decided in deterministic exponential time (ExpTime).