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8. Exercises for the Course 'Description Logics'

Exercise 27:

For each of the following languages of binary trees over the alphabet $\Sigma = \{a, b\}$, define a looping tree automaton that accepts the language:

(a) The set of all trees that contain a branch (starting at the root) in which all nodes are labelled with a;

(b) The set of all trees T that do not contain nodes n_0, n_1, n_2 such that (i) $n_1 = n_0 i$ for some $i \in \{0, 1\}$, (ii) $n_2 = n_1 j$ for some $j \in \{0, 1\}$, and (iii) $T(n_0) = T(n_1) = T(n_2) = a$.

Exercise 28:

Recall that a *propositional Horn clause* is a formula of the form $p_1, \ldots, p_k \to p$, where $p_1, \ldots, p_k$ are propositional letters and p is a propositional letter or $\bot$. A *Horn formula* is a set of Horn clauses. Also recall that the satisfiability of Horn formulas can be decided in linear time.

Show that the emptiness problem for looping tree automata can be decided in linear time by giving a linear-time reduction to the satisfiability of Horn formulas.

You may assume that the transition table is given in the form of a list of transitions such that transitions for the same state q on the left are grouped together and states without transitions are marked as such in the transition table.

Exercise 29:

Show that there is no looping tree automaton on binary $\{a, b\}$-trees that accepts the set of all trees that contain a branch with infinitely many nodes labeled with a.