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11. Exercises for the Course 'Description Logics'

Exercise 35:

Finish the proof of Lemma 6.7 by showing that $v_0 \in C_G^I$.

Exercise 36:

Determine whether or not Player 2 has a winning strategy in the PSPACE game $G = (\varphi, \{p_0, p_2\}, \{p_1, p_3\})$ with

$$\varphi = (\neg p_0 \to p_1) \wedge ((p_0 \wedge p_1) \to (p_2 \vee p_3)) \wedge (\neg p_1 \to (p_3 \to \neg p_2))$$

Exercise 37:

Determine whether or not Player 2 has a winning strategy in the EXPTIME game $G' = (\varphi, \Gamma_1, \Gamma_2, t_0)$ with

- $\varphi = (p_1 \wedge p_2 \wedge p_3 \wedge \neg q) \vee (\neg p_1 \wedge \neg p_2 \wedge \neg p_3 \wedge q)$,
- $\Gamma_1 = (p_1, p_2, p_3)$,
- $\Gamma_2 = (q)$,
- $t_0(p_1) = t_0(p_2) = t_0(p_3) = t_0(q) = 0$.

Exercise 38:

A *quantified Boolean formula* (QBF for short) Φ is of the form

$$Q_1 p_1 . Q_2 p_2 . \ldots Q_n p_n . \varphi$$

for $Q_i \in \{\forall, \exists\}$ and φ a Boolean formula over $p_1, \ldots, p_n$. The validity of QBFs is defined inductively:

$\exists p.\Phi$	is valid if	$\Phi[p/t]$ or $\Phi[p/f]$ is valid
$\forall p.\Phi$	is valid if	$\Phi[p/t]$ and $\Phi[p/f]$ are valid.

Reduce the problem of deciding the validity of QBFs to the problem of deciding the existence of a winning strategy for PSpace games.