# Selected Topics in Automata Theory

## Exercise Sheet 1

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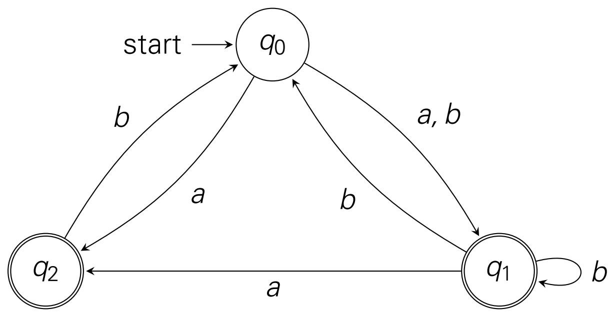
### Exercise 1

Let the non-deterministic automaton $\mathcal{A} := (\{q_0, q_1, q_2\}, \{a, b\}, \{q_0\}, \Delta, \{q_1, q_2\})$ be given by the following transition system.




Apply the power set construction to $\mathcal{A}$ in order to obtain a *deterministic* automaton that accepts the same language as $\mathcal{A}$.

### Exercise 2

Let $\Sigma = \{a, b\}$. Let $L_1$ be the language defined by

$$L_1 = \{w \in \Sigma^* \mid \text{ the number of occurrences of } a \text{ in } w \text{ is odd}\}$$

a) Give a regular expression $r$ such that $L(r) = L_1$.

b) Construct a finite automaton $\mathcal{A}$ such that $L_1 = L(\mathcal{A})$.

### Exercise 3

Let $\Sigma = \{a, b\}$. For each of the following regular expressions $r_i$ show that $L(r_i)$ is star-free by giving a star-free description of it (Example: $ab^* = a \cdot (\overline{\overline{\emptyset} \cdot a \cdot \overline{\emptyset}})$).

a) $\Sigma^*$

b) $aaa\Sigma^* + b^*$

c) $a^*b^* + b^*a^*$

d) $(abb^*)^*$

**Exercise 4**

For each of the languages $L(r_i)$ from Exercise 3 give a first-order formula $\varphi_i$ such that either $L(r_i) = L(\varphi_i)$ or $L(r_i) = L(\varphi_i) \cup \{\varepsilon\}$.