Selected Topics in Automata and Logic

Exercise Sheet 2

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Exercise 1

Let $\Sigma$ and $\Gamma$ be alphabets and $L \subseteq \Sigma^*$. Prove or refute the following implications:

a) $L \in SF_\Sigma \Rightarrow L \in SF_{\Sigma \cup \Gamma}$

b) $L \in SF_{\Sigma \cup \Gamma} \Rightarrow L \in SF_\Sigma$

Exercise 2

A finite one-way automaton $A = (Q, \Sigma, I, \Delta, F)$ is called a loop automaton if $Q = F$, i.e. all states are final states.

a) Prove that there cannot be a loop automaton that accepts the language $(aa)^*$.

b) Assume that we only allow inputs that start with $<$ and end with $>$, and that $>$ and $<$ may not occur in any other position. Give a loop automaton that accepts $>(aa)^*<$.

Exercise 3

Describe the languages $L(A_i)$, $i \in \{1, \ldots, 3\}$, that are accepted by the following two-way automata $A_i$ and give a regular expression for them.

a) $A_1 = (\{q_0, q_1\}, \{a, b\}, \{q_0\}, \Delta, \{q_0\})$

$\Sigma \cup \{>, 1\}$

$\Sigma \cup \{<\}, 1\}$

\begin{align*}
q_0 & \xrightarrow{(b, -1)} q_1 \\
q_1 & \xrightarrow{(a, -1)} q_0
\end{align*}
Exercise 4

Let $A = (Q, \Sigma, I, \Delta, F)$ be a deterministic, finite, one-way automaton that accepts the language $L$. Let

$$\overline{L} = \{a_n a_{n-1} \cdots a_1 | a_1 a_2 \cdots a_n \in L\}$$

be the language of all words from $L$ read backwards. Give a deterministic, finite, two-way automaton that accepts $\overline{L}$. 

b) $A_2 = (\{q_0, q_1\}, \{a, b, c\}, \{q_0\}, \Delta, \{q_1\})$

\[
\begin{align*}
&\{(\triangleright, c), 1\} \\
&\{b, \triangleleft\}, -1
\end{align*}
\]

\[
\xymatrix{ q_0 \ar[r]^{(a, 0)} & q_1 }
\]

c) $A_3 = (\{q_0, q_1, q_2\}, \{a, b\}, \{q_0\}, \Delta, \{q_2\})$

\[
\begin{align*}
&(b, -1) \\
&(\{a, \triangleright\}, 1)
\end{align*}
\]

\[
\xymatrix{ q_0 \ar[r]^{(b, 1)} & q_1 \\
& (q_2, (b, 1)) \ar[u]^{(<, 0)} &}
\]