Faculty of Computer Science Institute for Theoretical Computer Science, Chair for Automata Theory

Selected Topics in Automata and Logic

Exercise Sheet 2

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Exercise 1

Let Σ and Γ be alphabets and $L \subseteq \Sigma^*$. Prove or refute the following implications:

a) $L \in \mathbf{SF}_\Sigma \Rightarrow L \in \mathbf{SF}_{\Sigma\cup\Gamma}$

b) $L \in \mathbf{SF}_{\Sigma\cup\Gamma} \Rightarrow L \in \mathbf{SF}_\Sigma$

Exercise 2

A finite one-way automaton $\mathcal{A} = (Q, \Sigma, I, \Delta, F)$ is called a *looping automaton* if $Q = F$, i. e. all states are final states.

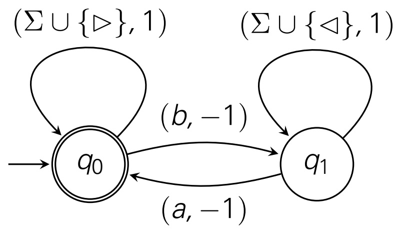
a) Prove that there cannot be a looping automaton that accepts the language $(aa)^*$.

b) Assume that we only allow inputs that start with $\rhd$ and end with $\lhd$, and that $\rhd$ and $\lhd$ may not occur in any other position. Give a looping automaton that accepts $\rhd(aa)^*\lhd$.

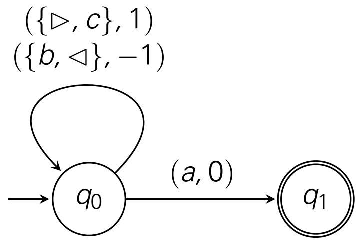
Exercise 3

Describe the languages $L(\mathcal{A}_i)$, $i \in \{1, \ldots, 3\}$, that are accepted by the following two-way automata $\mathcal{A}_i$ and give a regular expression for them.

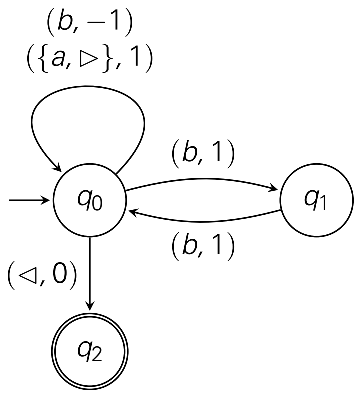
a) $\mathcal{A}_1 = (\{q_0, q_1\}, \{a, b\}, \{q_0\}, \Delta, \{q_0\})$



b) $\mathcal{A}_2 = (\{q_0, q_1\}, \{a, b, c\}, \{q_0\}, \Delta, \{q_1\})$



c) $\mathcal{A}_3 = (\{q_0, q_1, q_2\}, \{a, b\}, \{q_0\}, \Delta, \{q_2\})$



Exercise 4

Let $\mathcal{A} = (Q, \Sigma, I, \Delta, F)$ be a deterministic, finite, one-way automaton that accepts the language L. Let

$$\overleftarrow{L} = \{a_n a_{n-1} \cdots a_1 \mid a_1 a_2 \cdots a_n \in L\}$$

be the language of all words from L read backwards. Give a deterministic, finite, two-way automaton that accepts $\overleftarrow{L}$.