TECHNISCHE UNIVERSITÄT DRESDEN

Faculty of Computer Science Institute for Theoretical Computer Science, Chair for Automata Theory

Selected Topics in Automata and Logic

Exercise Sheet 3

Dr. Rafael Peñaloza / Dipl.-Math. Felix Distel
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Exercise 1

Construct two-way automata $\mathcal{A}_i$, $i \in \{1, \dots, 3\}$, that accept the following languages $\mathcal{L}_i$.

a) $L_1 \subseteq \{a, b\}^*$, $L_1 = a\Sigma^*$,

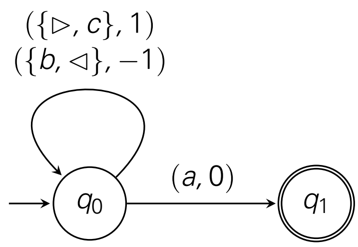
b) $L_2 \subseteq \{a, b\}^*$, $L_2 = \{w \mid \text{in } w \text{ every } b \text{ is preceded by an } a\}$,

c) $L_3 \subseteq \{a\}^*$, $L_3 = a(aa)^* \cup aa$.

Try not to use more states than necessary.

Exercise 2

Let $\mathcal{A} = (\{q_0, q_1\}, \{a, b, c\}, \{q_0\}, \Delta, \{q_1\})$ be the two-way finite state automaton that is defined by the following transition relation.



Using the construction from Theorem 2.4 from the lecture construct a non-deterministic one-way automaton $\bar{\mathcal{A}}$ that accepts the complement language $\overline{L(\mathcal{A})}$.

Exercise 3

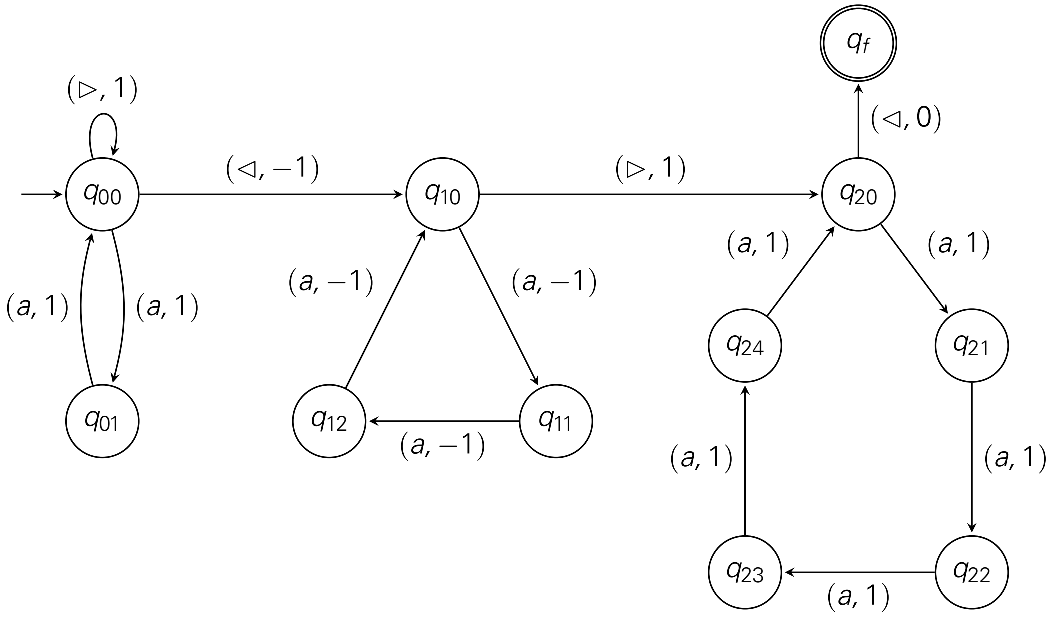
Provide for each of the following languages an MSO-formula as well as a formula using first-order logic with transitive closure.

a) $((ab)^2)^*$

b) $L = \{w \mid a \text{ occurs an even number of times in } w\}$

Exercise 4

Let $\mathcal{A} = (Q, \{a\}, \{q_{00}\}, \Delta, \{q_f\})$ with $Q = \{q_{00}, q_{01}, q_{10}, q_{11}, q_{20}, q_{21}, q_{22}, q_{23}, q_{24}, q_f\}$ be the *deterministic* two-way automaton defined by the following transition relation.



a) Give a regular expression for the language $L(\mathcal{A})$ that is accepted by $\mathcal{A}$.

b) What is the minimum number of states of a non-deterministic one-way automaton that accepts $L(\mathcal{A})$? Prove your claim.