Selected Topics in Automata and Logic

Exercise Sheet 4

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Summer Semester 2010

Exercise 1

As in Example 1.7 from the lecture words over an alphabet $\Sigma = \{a, b\}$ can be viewed as interpretations of a first-order formula. A formula $\varphi(x, y)$ from first-order logic with two free variables can be viewed as a binary relation over the domain of the interpretation (as described in Section 2.2).

Consider the word *aabba*. For the following formulae list all the pairs that are in the corresponding relation, as well as the pairs in the reflexive transitive closure, and the pairs in the deterministic transitive closure of that relation.

a) $\varphi_1(x, y) = \mathsf{Succ}(x, y)$

b) $\varphi_2(x, y) = \mathsf{Succ}(x, y) \wedge Q_b(x)$

c) $\varphi_3(x, y) = \exists z.Q_a(z)$

d) $\varphi_4(x, y) = Q_a(x) \wedge Q_b(y)$

Exercise 2

Let $\Sigma = \{a, b\}$. For each of the following formulae ψ_i, $i \in \{1, \ldots, 4\}$, give a regular expression for $L(\psi_i)$. The φ_i denote the formulae from Exercise 1.

a) $\psi_1 = \mathsf{TC}[\varphi_2](\min, \max)$

b) $\psi_2 = \mathsf{TC}[\varphi_3](\min, \max)$

c) $\psi_3 = \mathsf{DTC}[\varphi_3](\min, \max)$

d) $\psi_4 = \forall x \forall y.\big((\mathsf{TC}[\varphi_1](x, y) \wedge Q_b(x)) \Rightarrow Q_b(y)\big)$

Exercise 3

Let L_1, L_2 and L_3 be the languages over the alphabet $\Sigma = \{a, b\}$ defined by the following regular expressions.

a) aa

b) $abab^*$

c) a^+b^+

For each language L_i give a first order formula φ_i such that $L(\varphi_i) = L_i$. Using transitive closure construct a second formula ψ_i such that $L(\psi_i) = \Sigma \cdot L_i^*$.

Exercise 4

Show that the following languages are not regular (Hint: Prove indirectly that there cannot be a non-deterministic finite automaton that accepts them).

a) $L_1 = \{a^n b^n \mid n \in \mathbb{N}\}$

b) $L_2 = \{w\overleftarrow{w} \mid w \in \{a, b\}^*\}$