TECHNISCHE UNIVERSITÄT DRESDEN

Faculty of Computer Science Institute for Theoretical Computer Science, Chair for Automata Theory

Selected Topics in Automata and Logic

Exercise Sheet 5

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Definition

A *two-way automaton with k heads* is a tuple of the form $\mathcal{A} = (Q, \Sigma, I, \Delta, F)$, where Q, Σ, I and F are as in regular two-way automata, and $\Delta \subseteq Q \times (\Sigma \cup \{\triangleright, \triangleleft\})^k \times Q \times \{-1, 0, 1\}^k$ is the transition relation.

A *configuration* of a two-way k-head automaton $\mathcal{A}$ is an element of $Q \times \mathbb{N}^k$. A *run of $\mathcal{A}$ on a word* $w = a_1 \ldots a_n \in \Sigma^*$ is a sequence of configurations $(q_0, j_{01}, \ldots, j_{0k}) \ldots (q_m, j_{m1}, \ldots, j_{mk})$ such that

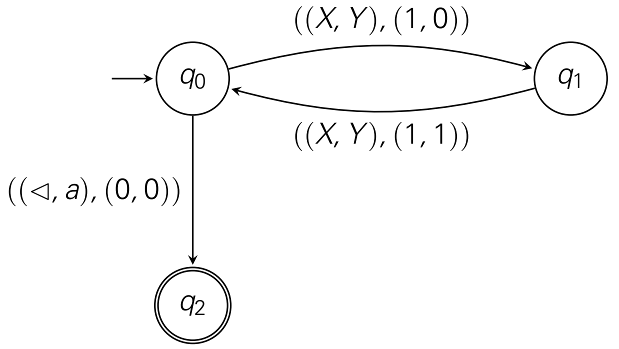
- $q_0 \in I$, and
- $j_{0\ell} = 0$ for all ℓ, $1 \leq \ell \leq k$, and
- $0 \leq j_{i\ell} \leq n + 1$ for all i, $0 \leq i \leq m$, and all ℓ, $1 \leq \ell \leq k$, and
- $(q_i, (a_{j_{i1}}, \ldots, a_{j_{ik}}), (j_{i+1,1} - j_{i1}, \ldots, j_{i+1,k} - j_{ik})) \in \Delta$ for all $i, 0 \leq i \leq m$.

A run is *accepting* if $q_m \in F$.

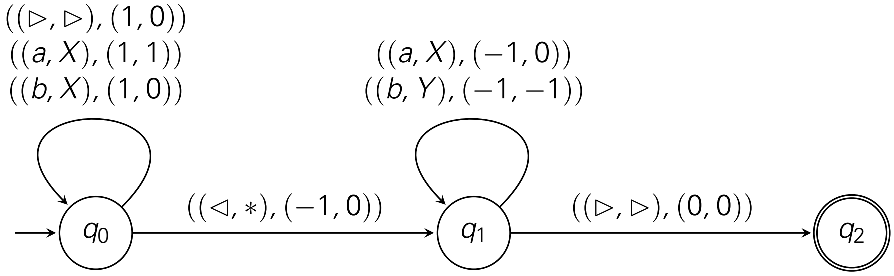
Exercise 1

Describe the languages that are accepted by the following two-way automata with 2 heads.

a) $\mathcal{A}_1 = (\{q_0, q_1, q_2\}, \{a, b\}, \{q_0\}, \Delta, \{q_2\})$, where Δ is given by the following relation (where X and Y range over all symbols from $\Sigma \cup \{\triangleright\}$).



b) $\mathcal{A}_1 = (\{q_0, q_1, q_2\}, \{a, b\}, \{q_0\}, \Delta, \{q_2\})$, where Δ is given by the following relation (where X ranges over all symbols from $\Sigma \cup \{\triangleright\}$, and Y ranges over all symbols from Σ).



Exercise 2

Give formal grammars for the languages from Exercise 1. What is the least Chomsky Type of these grammars?

Exercise 3

Construct two-way multihead automata that accept the following languages.

a) $L_1 = \{a^n b^n \mid n \in \mathbb{N}\}$

b) $L_2 = \{w\overleftarrow{w} \mid w \in \{a, b\}^*\}$

Exercise 4

Let $\varphi(x, y)$ be a first-order formula with 2 free variables. Let ψ be the formula $\psi(x, y) = \mathrm{TC}[\varphi](x, y)$ and let ψ' be the formula $\psi'(x, y) = \mathrm{DTC}[\psi](x, y)$. Give a regular expression for $L(\psi'(\min, \max))$. Prove your claim.