



## Selected Topics in Automata and Logic

### Exercise Sheet 8

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Summer Semester 2010

#### Exercise 1

Let  $G_2$ ,  $C_m$ ,  $D_m$ ,  $E_m$  and  $F_m$  be languages as defined in the lecture. Show that  $G_2 \cup C_m$  satisfies the conditions of Lemma 3.11, i. e.

- $C_m \cup D_m \subseteq G_2 \cup C_m$ , and
- $(G_2 \cup C_m) \cap (E_m \cup F_m) = \emptyset$

#### Exercise 2

A stateless multihead automaton is a multihead automaton with only one non-final state.

Let  $\mathcal{A}$  be a 2- $k$ FA. Is there a number  $k' \in \mathbb{N}$  such that  $L(\mathcal{A})$  can be accepted by a stateless 2- $k'$ FA?

#### Exercise 3

Let  $k \in \mathbb{N}$  be fixed and let  $\Sigma$  be a finite alphabet. Show that the class of languages over  $\Sigma$  that can be accepted by a one-way  $k$ -head non-deterministic finite automaton (1- $k$ FA) is

- a) closed under finite union, and
- b) *not* closed under intersection, and
- c) *not* closed under concatenation.

Hint: For proving Part b try to find two languages  $L$  and  $L'$  that can both be accepted by a 1- $k$ FA that satisfy  $L \cap L' = L_m$  for  $m = 1 + \binom{k}{2}$ .

For proving Part c you can use your result from Part a.