TECHNISCHE UNIVERSITÄT DRESDEN

Faculty of Computer Science Institute for Theoretical Computer Science, Chair for Automata Theory

Selected Topics in Automata and Logic

Exercise Sheet 10

Dr. Rafael Peñaloza / Dipl.-Math. Felix Distel
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Exercise 1

Let L_k be the set of all languagues that can be described using a first order logic with k-adic transitive closure. Prove or disprove the following statements.

a) L_k is closed under finite union.

b) L_k is closed under intersection.

c) L_k is closed under complement.

Exercise 2

Let $\Sigma = \{a\}$ be a unary alphabet. By $L_{2\text{-}k\text{FA}}$ denote the set of all languages that can be accepted by a 2-kFA. Prove or disprove the following statement. There is a number k_0 such that

$$\bigcup_{k \in \mathbb{N}} L_{2\text{-}k\text{FA}} = L_{2\text{-}k_0\text{FA}}.$$

Exercise 3

Give formulas from first order logic with k-adic transitive closure that describes the following languages.

- $L_1 = \{w\overleftarrow{w}wx \mid w \in \Sigma^*, x \in \Sigma\}$
- $L_2 = \overline{\{w\overleftarrow{w} \mid w \in \Sigma^*\}}$

Exercise 4

Give two formulas with two free variables $\varphi_1(a, b)$ and $\varphi_2(a, b)$ such that $\varphi_1(a, b)$ or $\varphi_2(a, b)$ is true iff the subword that starts at position a and ends at position b is in L_1 or L_2, respectively.

How can you use φ_1 and φ_2 to describe the languages $L_1 \cdot L_2$ and $L_2 \cdot L_1$?