TECHNISCHE UNIVERSITÄT DRESDEN

Faculty of Computer Science Institute for Theoretical Computer Science, Chair for Automata Theory

Selected Topics in Automata and Logic

Exercise Sheet 11

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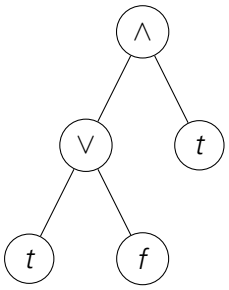
Exercise 1

a) Show that all finite tree languages are regular.

b) Show that all finite tree languages are recognizable by a tree walking automaton.

Exercise 2

Consider boolean expressions over the binary operators $\wedge$ and $\vee$ and the nullary symbols t and f. These expressions can be represented by trees, e. g. $b = (t \vee f) \wedge t$ can be represented by



Construct tree walking automata that accept the following tree languages over the alphabet $\Sigma = \{\wedge, \vee, t, f\}$:

a) the language of all trees that have at least one leaf that is labelled with f, and

b) the language of all trees that represent well formed boolean expressions, i. e. where all leafs are labelled with t or f and all internal nodes are labelled with $\vee$ or $\wedge$, and

c) the language of all trees that represent a boolean expression that evaluates to true.

For each of your automata give an accepting run for the tree representing b.

Exercise 3

Give FOL+TC formulae that describe the languages from Exercise 2.

Exercise 4

Let Σ be an alphabet. For each tree t over Σ we define a language $\mathrm{path}(t)$ as follows.

$$\mathrm{path}(t) := \{a_1 a_2 \dots a_n \in \Sigma^* \mid \text{there is a leaf } i_1 i_2 \dots i_n \text{ in } t \text{ such that } t(i_1 \dots i_k) = a_k \text{ for all } k \leq n\},$$

i. e. $\mathrm{path}(t)$ is the language of all words labelling a path in t. For a tree language B define

$$\mathrm{path}(B) := \bigcup_{t \in B} \mathrm{path}(t).$$

Conversely, for a language $L \subseteq \Sigma^*$ let

$$\mathrm{tree}(L) := \{t \in T_\Sigma \mid \mathrm{path}(t) \subseteq L\}.$$

Prove or refute.

a) $\mathrm{tree}(\mathrm{path}(B)) = B$

b) $\mathrm{path}(\mathrm{tree}(L)) = L$

c) If L is regular then $\overline{\mathrm{tree}(L)}$ is recognizable by a 1-head tree walking automaton.

d) If $\mathrm{path}(L)$ is regular then L is recognizable by a 1-head tree walking automaton.

Exercise 5

Let $\mathcal{L}_{1\mathrm{TWA}}$ be the set of all tree languages over a fixed alphabet Σ that can be recognized by a 1-head tree walking automaton. Prove or disprove.

a) $\mathcal{L}_{1\mathrm{TWA}}$ is closed under union.

b) $\mathcal{L}_{1\mathrm{TWA}}$ is closed under intersection.