Description Logics

Exercise Sheet 2

Dr. Anni-Yasmin Turhan / Dipl.-Math. Felix Distel
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Exercise 1

Extend the mapping τ_x of $\mathcal{ALC}$-concept descriptions to first-order formulas given in the lecture to the description logic $\mathcal{ALCQ}$, which augments $\mathcal{ALC}$ with qualified number restrictions.

Exercise 2

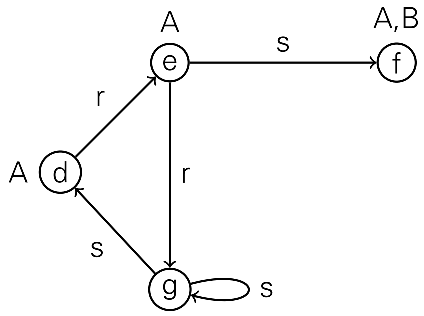
Recall that the description logic $\mathcal{ALC}$ is equipped with the concept constructors negation ($\neg$), conjunction ($\sqcap$), disjunction ($\sqcup$), existential restriction ($\exists r.C$), and universal restriction ($\forall r.C$). Each subset of this set of constructors gives rise to a fragment of $\mathcal{ALC}$.

Identify all minimal fragments that are equivalent to $\mathcal{ALC}$ in the sense that for every $\mathcal{ALC}$-concept, there is an equivalent concept in the fragment.

Two concepts are equivalent iff the concepts have the same extension in every interpretation.

Exercise 3

Consider the (graphical representation of the) interpretation $\mathcal{I}$ with $\Delta^{\mathcal{I}} = \{d, e, f, g\}$:



For each of the following $\mathcal{ALCNI}$-concepts C, list all elements x of $\Delta^{\mathcal{I}}$ such that $x \in C^{\mathcal{I}}$:

a) $A \sqcup B$

b) $\exists s.\neg A$

c) $\forall s.A$

d) $(\geq 2s)$

e) $\exists s.\exists s.\exists s.\exists s.A$

f) $\forall s^{-1}.\exists s.\exists s.\exists s.A$

g) $\neg\exists r.(\neg A \sqcap \neg B)$

h) $\exists s.(A \sqcap \forall s.\neg B) \sqcap \neg\forall r.\exists r.(A \sqcup \neg A)$

Exercise 4

Revisit the procedure for expanding TBoxes given in the proof of Proposition 2.6. Prove that

a) this procedure always terminates, and

b) that it returns a TBox that is equivalent to its input.

Hint for proving termination: count, for each concept name A, the number of concept names (directly or indirectly) used in the definition of A.

Exercise 5

Consider the TBox

$$\mathcal{T} = \{\neg(A \sqcup B) \sqsubseteq \bot, \quad A \sqsubseteq \neg B \sqcap \exists r.B, \quad D \sqsubseteq \forall r.A, \quad B \sqsubseteq \neg A \sqcap \exists r.A\},$$

the ABox

$$\mathcal{A} = \{r(a,b), \quad r(a,c), \quad r(a,d), \quad r(d,c), \quad (B \sqcap \forall r.D)(a), \quad E(b), \quad (\neg A)(c), \quad (\exists s.\neg D)(d)\}$$

and the knowledge base $\mathcal{K} = (\mathcal{T}, \mathcal{A})$. Check for

a) the TBox $\mathcal{T}$

b) the ABox $\mathcal{A}$ and

c) the knowledge base $\mathcal{K}$

whether it has a model. If it has one, specify such a model. If it does not have a model, explain why.