Faculty of Computer Science Institute of Theoretical Computer Science, Chair of Automata Theory

Description Logics

Exercise Sheet 4

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Summer Semester 2011

Exercise 1

In the lecture we saw that bisimulations allow to compare the expressive powers of DLs.

a) Extend the notion of bisimulation relation to $\mathcal{ALCN}$.

b) Prove that $\mathcal{ALCN}$ is bisimulation invariant for the bisimulation relation defined in (a).

c) Prove that $\mathcal{ALCQ}$ is more expressive than $\mathcal{ALCN}$.

Exercise 2

Let $\mathcal{K} = (\mathcal{T}, \mathcal{A})$ be a consistent knowledge base. We write $C \sqsubseteq_{\mathcal{K}} D$ if $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$ for all models $\mathcal{I}$of $\mathcal{K}$. Prove that for all $\mathcal{ALC}$-concepts C and D, we have $C \sqsubseteq_{\mathcal{K}} D$ iff $C \sqsubseteq_{\mathcal{T}} D$.

Hint: Use disjoint unions.

Exercise 3

Show the following claim:
If a concept C is satisfiable w.r.t. an $\mathcal{ALC}$-TBox $\mathcal{T}$, then for all $n \geq 1$ there is a model $\mathcal{I}_n$ of $\mathcal{T}$ such that: $|C^{\mathcal{I}_n}| \geq n$.

Exercise 4

Prove or refute the following claim:
Given an $\mathcal{ALC}$-concept C and an $\mathcal{ALC}$-TBox $\mathcal{T}$. If $\mathcal{I}$ is an interpretation and $\mathcal{J}$ its filtration w.r.t. Sub(C) $\cup$ Sub($\mathcal{T}$), then the relation $\varrho = \{(d, [d]) \mid d \in \Delta^{\mathcal{I}}, [d] \in \Delta^{\mathcal{J}},\ d \simeq [d]\}$ is a bisimulation.