TECHNISCHE UNIVERSITÄT DRESDEN

Faculty of Computer Science Institute of Theoretical Computer Science, Chair of Automata Theory

Description Logics

Exercise Sheet 7

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Exercise 1

We consider another form of blocking, where an individual can be blocked by an individual that is not necessarily an ancestor: anywhere blocking. Instead of the depth of an individual and the ancestor relation, it uses the age of an individual and the relation $<$.

The *age* of an individual x ($age(x)$) is defined as 0 for old individuals and n for a new individual x, if x was generated by the nth application of the $\exists$-rule.

Let $\mathcal{A}$ be an ABox obtained by applying the tableau rules and the GCI rule to an initial ABox $\mathcal{A}_0$. A new individual x is *anywhere blocked* by an individual a in an ABox $\mathcal{A}$, iff

- $\{C \mid C(x) \in \mathcal{A}\} \subseteq \{D \mid D(a) \in \mathcal{A}\}$, and
- $age(a) < age(x)$.

Prove the following for this form of blocking:

a) soundness

b) completeness
Hint: For what subset of the complete tableau do we need to construct a model?

c) termination

Exercise 2

For each of the following languages of binary trees over the alphabet $\Sigma = \{a, b\}$, define a looping tree automaton that accepts the language.

a) The set of all trees that contain a branch (starting at the root) in which all nodes are labelled with a.

b) The set of all trees that do not contain nodes n_0, n_1, n_2 such that

- $n_1 = n_0 i$ for some $i \in \{0, 1\}$,
- $n_2 = n_1 j$ for some $j \in \{0, 1\}$, and
- $T(n_0) = T(n_1) = T(n_2) = a$.

Exercise 3

Show that there is no looping tree automaton on binary $\{a, b\}$-trees that accepts the set of all trees that contain a branch with infinitely many nodes labeled with a.

Exercise 4

Reconsider the claim: for all $D \in S_{C,\mathcal{T}}$ we have $D \in R(u) \Rightarrow u \in D^{\mathcal{I}_R}$. Show the claim by induction on the structure of D for the missing cases:

- $D = D_1 \sqcup D_2$ and
- $D = \forall r.E$.