TECHNISCHE UNIVERSITÄT DRESDEN

Faculty of Computer Science Institute of Theoretical Computer Science, Chair of Automata Theory

Description Logics

Exercise Sheet 10

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Exercise 1

Determine whether or not Player 2 has a winning strategy in the PSPACE game $G = (\varphi, \{p_0, p_2\}, \{p_1, p_3\})$ with

$$\varphi = (\neg p_0 \rightarrow p_1) \land ((p_0 \land p_1) \rightarrow (p_2 \lor p_3)) \land (\neg p_1 \rightarrow (p_3 \rightarrow \neg p_2))$$

Exercise 2

A *quantified Boolean formula* (QBF for short) Φ is of the form

$$Q_1 p_1 . Q_2 p_2 . \dots Q_n p_n . \varphi$$

for $Q_i \in \{\forall, \exists\}$ and φ a Boolean formula over $p_1, \dots, p_n$. The validity of QBFs is defined inductively:

$$\begin{array}{lll} \exists p.\Phi & \text{is valid if} & \Phi[p/t] \text{ or } \Phi[p/f] \text{ is valid} \\ \forall p.\Phi & \text{is valid if} & \Phi[p/t] \text{ and } \Phi[p/f] \text{ are valid.} \end{array}$$

Reduce the problem of deciding the validity of QBFs to the problem of deciding the existence of a winning strategy for PSpace games.

Exercise 3

Let $\mathcal{K} = (\mathcal{A}_0, \mathcal{T})$ be an $\mathcal{ALC}$-knowledge base, with $\mathcal{T}$ a general TBox. The *precompletion* of $\mathcal{K}$ is the set of ABoxes $\mathcal{M}$ that is produced by the tableau algorithm when starting with the set of ABoxes $\{\mathcal{A}_0\}$ and exhaustingly applying all rules except the $\exists$-rule. Do the following:

a) Show that $\mathcal{K}$ is consistent iff there is an open $\mathcal{A} \in \mathcal{M}$ such that for all individual names a occurring in $\mathcal{A}$, the concept $C^a_{\mathcal{A}} := \bigsqcap_{C(a)\in\mathcal{A}} C$ is satisfiable w.r.t. $\mathcal{T}$.

   Hint: For the "if" direction, proceed as follows. The correctness of the tableau algorithm for $\mathcal{ALC}$ implies that, if $C^a_{\mathcal{A}}$ is satisfiable, then exhaustively applying (all!) rules to the set of ABoxes $\{\{C^a_{\mathcal{A}}(a)\}\}$ yields a set $\mathcal{M}'$ that contains an open and complete ABox. Show how to join all these ABoxes to obtain an open and complete tableau for $\mathcal{A}$ and conclude that $\mathcal{A}_0$ is consistent w.r.t. $\mathcal{T}$.

b) Use the result from (a) to prove that ABox consistency in $\mathcal{ALC}$ can be decided in deterministic exponential time (EXPTIME).