TECHNISCHE UNIVERSITÄT DRESDEN

Faculty of Computer Science Institute of Theoretical Computer Science, Chair of Automata Theory

Fuzzy Logic

Solutions to Exercise Sheet 2

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Exercise 4

(A7) *Case 2 ($\mathcal{V}(\varphi) \leq \mathcal{V}(\psi)$):* Non-decreasingness implies

$$\mathcal{V}(\chi) \otimes \mathcal{V}(\chi) \otimes \mathcal{V}(\chi) \otimes \mathcal{V}(\varphi) \leq \mathcal{V}(\psi).$$

Three consecutive applications of Lemma 2.2 yield

$$\mathcal{V}(\chi) \leq \big((\mathcal{V}(\psi) \Rightarrow \mathcal{V}(\varphi)) \Rightarrow \mathcal{V}(\chi) \big) \Rightarrow \mathcal{V}(\chi).$$

The claim follows from the above and the intermediate result that has been shown in the tutorial.

Exercise 5

a) $\mathrm{BL} \vdash \varphi \to (\psi \to \varphi)$ (Lemma 2.14 (1))
  $\mathrm{BL} \vdash \big(\varphi \to (\psi \to \varphi) \big) \to \big(\psi \to (\varphi \to \varphi) \big)$ (Lemma 2.14 (2))
  $\mathrm{BL} \vdash \psi \to (\varphi \to \varphi)$ (modus ponens)
  $\mathrm{BL} \vdash \big((\varphi \& \psi) \to \varphi \big) \to (\varphi \to \varphi)$ (substituting e. g. $(\chi \& \delta) \to \chi$ for ψ)
  $\mathrm{BL} \vdash (\chi \& \delta) \to \chi$ (A2)
  $\mathrm{BL} \vdash \varphi \to \varphi$ (modus ponens)

b) $\mathrm{BL} \vdash \big(\varphi \& (\psi \& \chi) \big) \to \big(\varphi \& (\psi \& \chi) \big)$ (Exercise 5 a))
  $\mathrm{BL} \vdash \Big(\big(\varphi \& (\psi \& \chi) \big) \to \big(\varphi \& (\psi \& \chi) \big) \Big) \to \Big(\varphi \to \big((\psi \& \chi) \to (\varphi \& (\psi \& \chi)) \big) \Big)$ (A6)
  $\mathrm{BL} \vdash \varphi \to \Big((\psi \& \chi) \to (\varphi \& (\psi \& \chi)) \Big)$ (modus ponens)
  $\mathrm{BL} \vdash \Big((\psi \& \chi) \to (\varphi \& (\psi \& \chi)) \Big) \to \Big(\psi \to \big(\chi \to (\varphi \& (\psi \& \chi)) \big) \Big)$ (A6)
  $\mathrm{BL} \vdash \varphi \to \Big(\psi \to \big(\chi \to (\varphi \& (\psi \& \chi)) \big) \Big)$ ((A1) + modus ponens)
  $\mathrm{BL} \vdash (\varphi \& \psi) \to \Big(\chi \to (\varphi \& (\psi \& \chi)) \Big)$ ((A5) + modus ponens)
  $\mathrm{BL} \vdash \big((\varphi \& \psi) \& \chi \big) \to \big(\varphi \& (\psi \& \chi) \big)$ ((A5) + modus ponens)