TECHNISCHE UNIVERSITÄT DRESDEN

Faculty of Computer Science Institute of Theoretical Computer Science, Chair of Automata Theory

Fuzzy Logic

Solutions to Exercise Sheet 3

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Exercise 3

b) This is not a residuated lattice since

$$A \leq (B \,/\, C), \quad (\text{i.e. } A \supseteq C \setminus B)$$

does not imply

$$A \Delta B \leq C, \quad (\text{i.e. } A \Delta B \supseteq C).$$

As a counterexample consider $M = \mathbb{Z}$, $A = \{1, 2\}$, $B = \{2\}$, $C = \{1, 2\}$. Then

$$A = \{1, 2\} \supseteq \{1\} = C \setminus B$$

holds, but

$$A \Delta B = \{1\} \not\supseteq \{1, 2\} = C.$$