TECHNISCHE UNIVERSITÄT DRESDEN

Faculty of Computer Science Institute of Theoretical Computer Science, Chair of Automata Theory

Fuzzy Logic

Solutions to Exercise Sheet 4

Dr. Rafael Peñaloza Nyssen / Dipl.-Math. Felix Distel
Summer Semester 2011

Exercise 2

a) Let $x_1 \otimes x_2 \in F_{1\otimes 2}$, $x_1 \in F_1$, $x_2 \in F_2$ and let $x_1 \otimes x_2 \leq y$ for some $y \in L$. Since L is linearly ordered either $x_1 \leq x_2$ or $x_2 \leq x_1$. Assume w.l.o.g. $x_1 \leq x_2$. Since F_1 is a filter this yields $x_2 \in F_1$ and therefore $x_1 \otimes x_2 \in F_1$. Applying the definition of filters again yields $y \in F_1$. From $1 \in F_2$ we obtain $y = y \otimes 1 \in F_{1\otimes 2}$.

c) Let $f_1, g_1 \in F_1$, $f_2, g_2 \in F_2$. We show that $(f_1 \vee f_2) \otimes (g_1 \vee g_2) \in F_{1\vee 2}$. Filter properties of F_1 and F_2 yield $f_1 \otimes g_1 \in F_1$ and $f_2 \otimes g_2 \in F_2$ and thus $(f_1 \otimes g_1) \vee (f_2 \otimes g_2) \in F_{1\vee 2}$. On the other hand we obtain

$$(f_1 \vee f_2) \otimes (g_1 \vee g_2) \geq f_1 \otimes g_1$$
$$(f_1 \vee f_2) \otimes (g_1 \vee g_2) \geq f_2 \otimes g_2$$

from the monotonicity of $\otimes$. Hence it also holds that

$$(f_1 \vee f_2) \otimes (g_1 \vee g_2) \geq (f_1 \otimes g_1) \vee (f_2 \otimes g_2).$$

From the other filter property for $F_{1\vee 2}$ (already shown in the tutorial) and $(f_1 \otimes g_1) \vee (f_2 \otimes g_2) \in F_{1\vee 2}$ we obtain $(f_1 \vee f_2) \otimes (g_1 \vee g_2) \in F_{1\vee 2}$.