TECHNISCHE UNIVERSITÄT DRESDEN

Faculty of Computer Science Institute of Theoretical Computer Science, Chair of Automata Theory

Fuzzy Logic

Solutions to Exercise Sheet 4

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Summer Semester 2011

Exercise 1

a) $\Pi \vdash ((\varphi \to \neg\varphi)\&\varphi) \to \mathbf{0}$ (Π2) + (A3)
$\Pi \vdash (\varphi \to \neg\varphi) \to (\varphi \to \mathbf{0})$ (A6)
$\Pi \vdash (\varphi \to (\varphi \to \mathbf{0})) \to \neg\varphi$ def. of $\neg$
$\Pi \vdash ((\varphi\&\varphi) \to \mathbf{0}) \to \neg\varphi$ (A6) + (A1)
$\Pi \vdash \neg(\varphi\&\varphi) \to \neg\varphi$ def. of $\neg$

c) For the first direction it only remains to show that $\Pi \vdash \neg\neg\neg\varphi \to \neg\varphi$. From the properties of negation in BL we obtain
$\Pi \vdash \varphi \to \neg\neg\varphi$, and
$\Pi \vdash (\varphi \to \neg\neg\varphi) \to (\neg\neg\neg\varphi \to \neg\varphi)$, and thus
$\Pi \vdash \neg\neg\neg\varphi \to \neg\varphi$.

Consider the other direction where we want to prove $\Pi''' \vdash (\varphi \wedge \neg\varphi) \to \mathbf{0}$.
$\Pi''' \vdash \neg\neg\varphi \to ((\varphi\&\varphi \to \underbrace{\varphi\&\mathbf{0}}_{\equiv\mathbf{0}}) \to \underbrace{(\varphi \to \mathbf{0})}_{\neg\varphi})$, ($\Pi$1)
$\Pi''' \vdash \neg\varphi \to ((\varphi\&\varphi \to \mathbf{0}) \to \neg\varphi)$, (L. 2.14 (1))
$\Pi''' \vdash (\neg\varphi \vee \neg\neg\varphi) \to ((\varphi\&\varphi \to \mathbf{0}) \to \neg\varphi)$, (properties of weak disjunction)
$\Pi''' \vdash ((\varphi\&\varphi \to \mathbf{0}) \to \neg\varphi)$, c) + mod. pon.
The rest follows from a).