



Fuzzy Logic

Exercise Sheet 1

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Exercise 1

Show that the following three binary operators are continuous t-norms:

- Lukasiewicz t-norm*: $x \otimes y = \max\{x + y - 1, 0\}$,
- Product t-norm*: $x \otimes y = x \cdot y$,
- Gödel t-norm*: $x \otimes y = \min\{x, y\}$.

Exercise 2

A partial order on the set of all t-norms can be defined naturally as follows. Let \otimes_1 and \otimes_2 denote two t-norms. We write

$$\otimes_1 \leq \otimes_2 :\Leftrightarrow \forall u, v \in [0, 1] : u \otimes_1 v \leq u \otimes_2 v.$$

Find two t-norms \otimes_{\min} and \otimes_{\max} such that every t-norm \otimes satisfies $\otimes_{\min} \leq \otimes \leq \otimes_{\max}$.

Exercise 3

Prove Lemma 2.2 from the lecture.

Exercise 4

Show that for every continuous t-norm and its residuum \Rightarrow , and every $x, y \in [0, 1]$

- $x \leq y$ iff $x \Rightarrow y = 1$,
- $(1 \Rightarrow x) = x$.