TECHNISCHE UNIVERSITÄT DRESDEN

Faculty of Computer Science Institute of Theoretical Computer Science, Chair of Automata Theory

Fuzzy Logic

Exercise Sheet 2

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Exercise 1

Find the precomplement of the three continuous t-norms: Łukasiewicz t-norm, Product t-norm and Gödel t-norm.

Exercise 2

Using ordinal sums, construct a continuous t-norm where exactly 3 values from $[0, 1]$ are idempotent, i.e. exactly 3 values satisfy $x \otimes x = x$.

Exercise 3

Check for which of the above t-norms (if any) the following formulas are 1-tautologies.

a) $\neg\neg\varphi \to \varphi$

b) $\varphi \vee \neg\varphi$

c) $(\varphi \wedge \psi) \to (\varphi \& \psi)$

Exercise 4

Prove that the axioms (A5)-(A8) are 1-tautologies of $PC(\otimes)$.

(A5) $(\varphi \to (\psi \to \chi)) \to ((\varphi \& \psi) \to \chi)$

(A6) $((\varphi \& \psi) \to \chi) \to (\varphi \to (\psi \to \chi))$

(A7) $((\varphi \to \psi) \to \chi) \to (((\varphi \to \psi) \to \chi) \to \chi)$

(A8) $\mathbf{0} \to \varphi$

Exercise 5

Using only modus ponens and the axioms (A1)-(A8) prove the following formulas in BL.

a) $\varphi \to \varphi$

b) $((\varphi \& \psi) \& \chi) \to (\varphi \& (\psi \& \chi))$