TECHNISCHE UNIVERSITÄT DRESDEN

Faculty of Computer Science Institute of Theoretical Computer Science, Chair of Automata Theory

Fuzzy Logic

Exercise Sheet 4

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Exercise 1

Construct a BL-algebra such that the carrier set can be partitioned into $A \cup B \cup \{0, 1\}$ with the following properties:

- $\otimes$ is isomorphic to the Łukasiewicz t-norm when restricted to A,
- $\otimes$ is isomorphic to the product t-norm when restricted to B, and
- $x \otimes y = 0$ if $x \in A$ and $y \in B$ or $x \in B$ and $y \in A$.

Exercise 2

Let F_1 and F_2 be two filters on a linearly ordered BL-algebra **L**. Prove or disprove the following properties.

a) $\{f_1 \otimes f_2 \mid f_1 \in F_1, f_2 \in F_2\}$ is a filter,

b) $\{f_1 \wedge f_2 \mid f_1 \in F_1, f_2 \in F_2\}$ is a filter, and

c) $\{f_1 \vee f_2 \mid f_1 \in F_1, f_2 \in F_2\}$ is a filter.

Exercise 3

Let **L** be a countable BL-algebra and F a filter on **L**. Let $z \in L$ be an element. Show that

$$F' := \{u \mid \exists v \in F \colon \exists n \in \mathbb{N} \colon v \otimes z^n \leq u\}$$

is the least filter containing F and z.

Exercise 4

Prove the following result: For every theory T, if $T \nvdash \varphi$ then there exists a consistent complete supertheory $T' \supseteq T$ such that $T' \nvdash \varphi$.