Fuzzy Logic

Exercise Sheet 5

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Exercise 1

The Łukasiewicz axioms are

(Ł1) $\varphi \to (\psi \to \varphi)$,

(Ł2) $(\varphi \to \psi) \to ((\psi \to \chi) \to (\varphi \to \chi))$,

(Ł3) $(\neg\varphi \to \neg\psi) \to (\psi \to \varphi)$,

(Ł4) $((\varphi \to \psi) \to \psi) \to ((\psi \to \varphi) \to \varphi)$,

Show that

a) Ł, as introduced in the lecture, proves (Ł1), (Ł2), (Ł3), and (Ł4).

b) (Ł1), (Ł2), (Ł3), and (Ł4) prove $\neg\neg\varphi \to \varphi$.

Exercise 2

Let **A** be an MV-algebra and T a theory. An **A**-evaluation μ is called an **A**-*model* of T iff μ evaluates each formula $\varphi \in T$ to 1.

By $[0, 1]_{Ł}$ we denote the standard MV-algebra on $[0, 1]$ defined by the truth functions of Łukasiewicz logic.

Define

$$T = \{np \to q \mid n \in \mathbb{N}\} \cup \{\neg p \to q\}$$

where $np = \neg(\neg p \& \neg(n-1)p)$. Prove that q is true in all $[0, 1]_{Ł}$-models of T but this does not hold for any finite subset of T.