TECHNISCHE UNIVERSITÄT DRESDEN

Faculty of Computer Science Institute of Theoretical Computer Science, Chair of Automata Theory

Fuzzy Logic

Exercise Sheet 6

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Exercise 1

Show that in the product logic Π the axiom ($\Pi 2$) can be equivalently replaced by any of the following formulae:

a) $\neg(\varphi \,\&\, \varphi) \to \neg\varphi$,

b) $(\varphi \to \neg\varphi) \to \neg\varphi$, or

c) $\neg\varphi \vee \neg\neg\varphi$.

Exercise 2

Prove that the following sentences hold in every linearly ordered product algebra:

a) if $x > 0$ then $\ominus x = 0$,

b) if $z > 0$ then $x \otimes z = y \otimes z$ implies $x = y$, and

c) if $z > 0$ then $x \otimes z < y \otimes z$ implies $x < y$.

Exercise 3

Show that every linearly ordered MV-algebra is an algebra of the form $MV'(\mathbf{L}, a)$ where $\mathbf{L}$ is a linearly ordered product algebra, $a \in \mathbf{L}$ with $0_{\mathbf{L}} < a < 1_{\mathbf{L}}$, the domain of $MV'(\mathbf{L}, a)$ is the interval $[a, 1_{\mathbf{L}}]$.

Exercise 4

Show $\Pi \vdash (\varphi \,\&\, \varphi \to \mathbf{0}) \to (\varphi \to \mathbf{0})$.