TECHNISCHE UNIVERSITÄT DRESDEN

Faculty of Computer Science Institute of Theoretical Computer Science, Chair of Automata Theory

Automata and Logic

Exercise Sheet 4

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Exercise 17

Let V be an M-variety. Show that $L(V)_\Sigma$ is closed under union *without* using Thm. 1.22 from the lecture.

Exercise 18

Let Σ be an alphabet. Prove or refute the following claims:

a) Every regular language $L \subseteq \Sigma^*$ is accepted by its syntactic monoid.

b) If $L \subseteq \Sigma^*$ is accepted by a finite group, then the syntactic monoid of L is a finite group.

c) For every regular language $L \subseteq \Sigma^*$, the syntactic monoid M_L is the smallest monoid accepting L; i.e. for every monoid M accepting L, we have $|M_L| \leq |M|$.

d) For a word $w = a_1 \dots a_n$, let $\overleftarrow{w}$ denote the mirror image of w, i.e. $\overleftarrow{w} = a_n \dots a_1$. For a language $L \subseteq \Sigma^*$, we define $\overleftarrow{L} := \{\overleftarrow{w} \mid w \in L\}$. **Claim:** If the minimal automaton for L has n states, then the minimal automaton for $\overleftarrow{L}$ has also n states.

Exercise 19

Let V be the M-variety of all commutative finite groups. Show that there exists a language $L \subseteq \{a\}^*$ such that $L \in L(V)_{\{a\}}$ but $L \notin L(V)_{\{a,b\}}$.

Exercise 20

Prove or refute the following: There is a language $L \subseteq \{a, b\}^*$ such that its syntactic semigroup S_L and its syntactic monoid M_L are isomorphic.

Exercise 21

For each of the following words over the alphabet $\{0, 1\}^k$, give a corresponding interpretation over the predicate symbols $P_1, \dots, P_k$ as discussed in the lecture:

$$\begin{aligned} k = 2&: \quad (1,1),(1,1),(0,1),(1,0) \\ k = 3&: \quad (0,0,0),(1,0,0),(0,1,0),(0,0,1) \\ k = 3&: \quad (1,1,0),(1,0,1),(1,1,1),(1,1,0) \end{aligned}$$

Describe all interpretations that correspond to words of the language
$L(((0,1) \cdot (1,0))^+) \subseteq (\{0,1\}^2)^+$.

Exercise 22

Let $\Sigma = \{a, b\}$. For each of the following regular expressions r_i, give a first-order formula ϕ_i such that $L(r_i) = L(\phi_i)$.

a) $r_1 = \Sigma^*$,

b) $r_2 = \varepsilon$,

c) $r_3 = (abb^*)^*$,

d) $r_4 = a^*b^* + b^*a^*$, and

e) $r_5 = (aaa \cdot \Sigma^*) + b^*$.