Automata and Logic

Exercise Sheet 6

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Exercise 29

Let Σ be an alphabet. A language $L \subseteq \Sigma^*$ is called *definite* for Σ if there exists an $n \in \mathbb{N}$ such that we have for all $w \in L$:

$$\text{if } w = uv \text{ with } |u| = n, \text{ then } u\Sigma^* \subseteq L.$$

Show that $L \subseteq \Sigma^*$ is definite for Σ iff L is a Boolean combination of languages of the form $\{w\Sigma^* \mid w \in \Sigma^*\}$.

Exercise 30

Let $\Sigma = \{0, 1\}^k$. Show that the following is equivalent:

- L is definite for Σ.
- There exists a quantifier-free closed first-order formula ϕ over the signature $\{P_1, \ldots, P_k, <, \min, s\}$ with $L(\phi) = L \setminus \{\varepsilon\}$.

Exercise 31

Let Σ, Γ be two alphabets, and let $L \subseteq \Sigma^*$. Prove or refute the following:

a) $L \in \mathrm{SF}_\Sigma \implies L \in \mathrm{SF}_{\Sigma\cup\Gamma}$

b) $L \in \mathrm{SF}_{\Sigma\cup\Gamma} \implies L \in \mathrm{SF}_\Sigma$

Exercise 32

For $\Sigma = \{a, b\}$, check whether the following languages are star-free:

a) $L_1 = (ab)^*$

b) $L_2 = \{w \mid |w|_a = 3k \text{ for some } k \in \mathbb{N}\}$

c) $L_3 = a(aba)^*b$

Use Thm. 3.6 from the lecture or give a star-free description of the language.