Automata and Logic

Exercise Sheet 8

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Exercise 37

Let $\Sigma = \{a, b\}$, and $L \subseteq \Sigma^*$ be defined by the regular expression $(a^*bb^*)^*$. Show that

$$\lim L = \{\alpha \in \Sigma^\omega \mid \text{if } \alpha(i,i) = a, \text{ then there is a } j > i \text{ with } \alpha(j,j) = b\}.$$

Exercise 38

Give Büchi automata that recognise the following ω-regular languages over the alphabet $\Sigma := \{a, b, c\}$:

a) $L_1 := \{\alpha \in \Sigma^\omega \mid \exists i \in \mathbb{N} \colon \alpha(i, i+2) = abc\}$;

b) $L_2 := \{\alpha \in \Sigma^\omega \mid \{i \in \mathbb{N} \mid \alpha(i, i+2) = abc\} \text{ is infinite}\}$; and

c) $L_3 := (a^+b^+c^+)^\omega$.

Exercise 39

a) Show that the construction used in the proof of Lemma 4.7.1 does not work for automata whose initial state is reachable from another state.

b) Complete the proof of Lemma 4.7 from the lecture by showing the following:

$$\text{If } L_1, L_2 \subseteq \Sigma^\omega \text{ are Büchi recognisable, then } L_1 \cup L_2 \text{ is Büchi recognisable.}$$

Exercise 40

Let Σ be an alphabet, and $L, L_1, L_2 \subseteq \Sigma^*$. Prove or refute:

a)
- $(L_1 \cup L_2)^\omega \subseteq L_1^\omega \cup L_2^\omega$
- $(L_1 \cup L_2)^\omega \supseteq L_1^\omega \cup L_2^\omega$

b)
- $\lim(L_1 \cup L_2) \subseteq \lim L_1 \cup \lim L_2$
- $\lim(L_1 \cup L_2) \supseteq \lim L_1 \cup \lim L_2$

c)
- $L^\omega \subseteq \lim L^+$
- $L^\omega \supseteq \lim L^+$

d)
- $\lim(L_1 \cdot L_2) \subseteq L_1 \cdot L_2^\omega$
- $\lim(L_1 \cdot L_2) \supseteq L_1 \cdot L_2^\omega$