TECHNISCHE UNIVERSITÄT DRESDEN

Faculty of Computer Science Institute of Theoretical Computer Science, Chair of Automata Theory

Automata and Logic

Exercise Sheet 9

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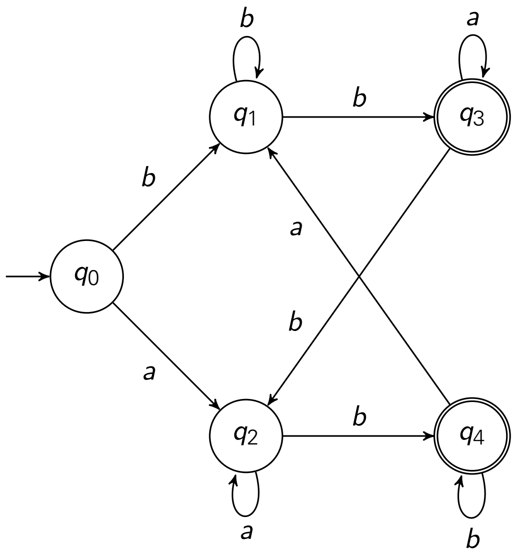
Exercise 41

Let $\Sigma = \{a, b\}$, and $L \subseteq \Sigma^\omega$ be the ω-language recognised by the following Büchi automaton:



Find a number $n \geq 1$ and regular languages $U_1, V_1, \ldots, U_n, V_n \subseteq \Sigma^*$ such that

$$\bigcup_{i=1}^{n} U_i \cdot V_i^\omega = L.$$

Exercise 42

Let Σ be an alphabet. Prove the following:

a) If $L \subseteq \Sigma^+$ is regular, then there exists a non-deterministic finite automaton $\mathcal{A}$ with only *one* final state such that $L = L(\mathcal{A})$.

b) If $L \subseteq \Sigma^*$ is regular, then there exists a non-deterministic finite automaton $\mathcal{A}$ with at most *two* final states such that $L = L(\mathcal{A})$.

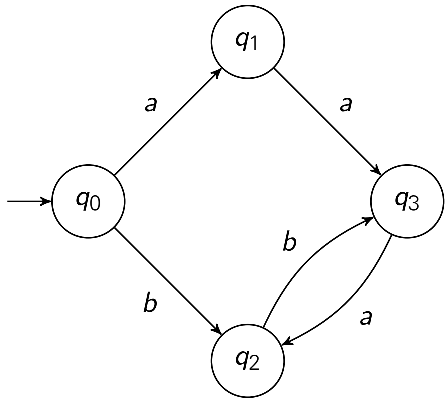
c) There is *no* $k \geq 1$ such that the following holds:

   If $L \subseteq \Sigma^\omega$ is Büchi recognisable, then there exists a Büchi automaton $\mathcal{A}$ with at most k final states such that $L = L_\omega(\mathcal{A})$.

   Hint: Consider the languages $a^\omega \cup b^\omega$, $a^\omega \cup b^\omega \cup c^\omega$, ...

Exercise 43

Consider Büchi automata using the following transition system:



Check whether the recognised ω-language is empty for the following sets of final states:

a) $F = \{q_0, q_1\}$

b) $F = \{q_2, q_3\}$

c) $F = \{q_1, q_3\}$

Exercise 44

For a finite automaton $\mathcal{A}$, let $\mathcal{A}_{\text{det}}$ denote the minimal deterministic finite automaton such that $L(\mathcal{A}) = L(\mathcal{A}_{\text{det}})$. Prove or refute the following:

a) $\lim L(\mathcal{A}) = L_\omega(\mathcal{A}_{\text{det}})$

b) $L_\omega(\mathcal{A}) \subseteq L_\omega(\mathcal{A}_{\text{det}})$

c) $L_\omega(\mathcal{A}_{\text{det}}) \subseteq L_\omega(\mathcal{A})$

Exercise 45

Let $(r_n)_{n\geq 0}$ be a sequence of real numbers. Show that there exists an infinite sub-sequence of $(r_n)_{n\geq 0}$ that is

- strictly increasing, or
- strictly decreasing, or
- constant.