Automata and Logic

Exercise Sheet 9

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Exercise 41

Let $\Sigma = \{a, b\}$, and $L \subseteq \Sigma^\omega$ be the $\omega$-language recognised by the following Büchi automaton:

![Büchi Automaton Diagram]

Find a number $n \geq 1$ and regular languages $U_1, V_1, \ldots, U_n, V_n \subseteq \Sigma^*$ such that

$$\bigcup_{i=1}^{n} U_i \cdot V_i^\omega = L.$$ 

Exercise 42

Let $\Sigma$ be an alphabet. Prove the following:

a) If $L \subseteq \Sigma^+$ is regular, then there exists a non-deterministic finite automaton $\mathcal{A}$ with only one final state such that $L = L(\mathcal{A})$.

b) If $L \subseteq \Sigma^*$ is regular, then there exists a non-deterministic finite automaton $\mathcal{A}$ with at most two final states such that $L = L(\mathcal{A})$.

c) There is no $k \geq 1$ such that the following holds:

If $L \subseteq \Sigma^\omega$ is Büchi recognisable, then there exists a Büchi automaton $\mathcal{A}$ with at most $k$ final states such that $L = L_\omega(\mathcal{A})$.

Hint: Consider the languages $a^\omega \cup b^\omega, a^\omega \cup b^\omega \cup c^\omega, \ldots.$
**Exercise 43**

Consider Büchi automata using the following transition system:

![Transition System Diagram]

Check whether the recognised $\omega$-language is empty for the following sets of final states:

a) $F = \{q_0, q_1\}$

b) $F = \{q_2, q_3\}$

c) $F = \{q_1, q_3\}$

**Exercise 44**

For a finite automaton $\mathcal{A}$, let $\mathcal{A}_{det}$ denote the minimal deterministic finite automaton such that $L(\mathcal{A}) = L(\mathcal{A}_{det})$. Prove or refute the following:

a) $\lim L(\mathcal{A}) = L_\omega(\mathcal{A}_{det})$

b) $L_\omega(\mathcal{A}) \subseteq L_\omega(\mathcal{A}_{det})$

c) $L_\omega(\mathcal{A}_{det}) \subseteq L_\omega(\mathcal{A})$

**Exercise 45**

Let $(r_n)_{n \geq 0}$ be a sequence of real numbers. Show that there exists an infinite sub-sequence of $(r_n)_{n \geq 0}$ that is

- strictly increasing, or
- strictly decreasing, or
- constant.