TECHNISCHE UNIVERSITÄT DRESDEN

**Faculty of Computer Science** Institute of Theoretical Computer Science, Chair of Automata Theory

# Automata and Logic

**Exercise Sheet 10**

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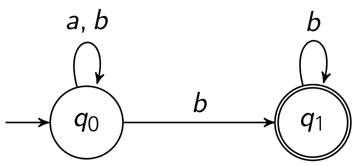
## Exercise 46

Let $\Sigma = \{a, b\}$, and $L \subseteq \Sigma^\omega$ be the $\omega$-language recognised by the following Büchi automaton:



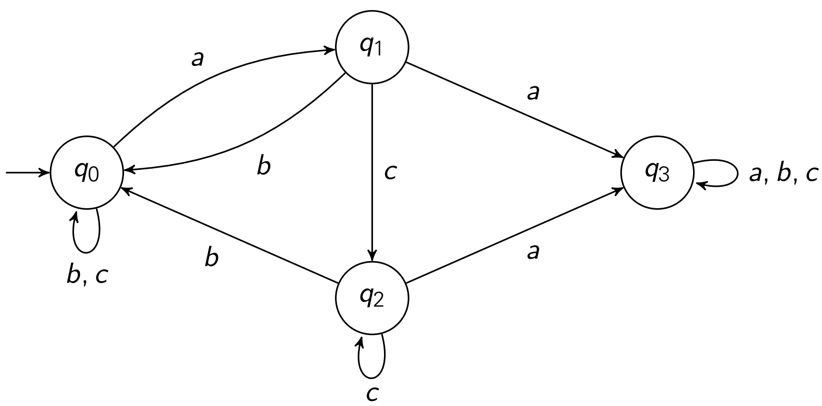

Use the method presented in the lecture to construct a Büchi automaton that recognises the language $\Sigma^\omega \setminus L$.

## Exercise 47

Prove that for every $\omega$-regular language $L$, there is a Büchi automaton $\mathcal{A}$ with $L_\omega(\mathcal{A}) = L$ such that from every state $q$ of $\mathcal{A}$, there are *at most two* transitions using the same alphabet symbol.

## Exercise 48

Let $\Sigma := \{a, b, c\}$. Consider the following transition system:




We derive four Muller automata $\mathcal{A}_1$, $\mathcal{A}_2$, $\mathcal{A}_3$, and $\mathcal{A}_4$ by selecting the sets of final states $\mathcal{F}_1$, $\mathcal{F}_2$, $\mathcal{F}_3$, and $\mathcal{F}_4$ as follows:

a) $\mathcal{F}_1 := \{\{q_0, q_3\}, \{q_3\}\}$;

b) $\mathcal{F}_2 := \{\{q_0, q_1\}, \{q_2\}\}$;

c) $\mathcal{F}_3 := \{\{q_0, q_1, q_2\}\}$; and

d) $\mathcal{F}_4 := \{\{q_0\}, \{q_0, q_1\}, \{q_2\}, \{q_0, q_1, q_2\}\}$.

Determine the $\omega$-languages $L_\omega(\mathcal{A}_1)$, $L_\omega(\mathcal{A}_2)$, $L_\omega(\mathcal{A}_3)$, and $L_\omega(\mathcal{A}_4)$.

**Exercise 49**

For each of the following languages $L_i$, give an S1S-formula $\phi_i$ such that $L_\omega(\phi_i) = L_i$:

a) $L_1 = (abb^*)^\omega$,

b) $L_2 = ((aa)^+(bb)^+)^\omega$, and

c) $L_3 = (aaa)^+b(a \cup b)^\omega$.

**Exercise 50**

Transform the S1S-formula $P(\underline{0})$ into an equivalent $\mathrm{S1S}_0$-formula.

**Exercise 51**

Let $L := (a^+b)^\omega \cup (b^+a)^\omega$. Use the proof of Thm. 5.4 to construct a closed S1S-formula $\phi$ with $L_\omega(\phi) = L$.

**Exercise 52**

A *Rabin automaton* is a tuple $\mathcal{A} := (Q, \Sigma, I, \Delta, \Omega)$ where $Q$, $\Sigma$, $I$, and $\Delta$ are defined as for non-deterministic Büchi automata, and $\Omega := \{(F_1, G_1), \dots, (F_n, G_n)\}$ is a finite set of pairs $(F_i, G_i)$ such that $F_i, G_i \subseteq Q$. For a word $\alpha$, let $\mathsf{path}_\mathcal{A}(\alpha)$ denote the set of all paths in $\mathcal{A}$ labelled with $\alpha$. For a path $p \in \mathsf{path}_\mathcal{A}(\alpha)$, let $\mathsf{inf}(p)$ denote the set of all states that are visited infinitely often. The $\omega$-language $L_\omega(\mathcal{A})$ recognised by a Rabin automaton is defined as

$$L_\omega(\mathcal{A}) := \{\alpha \in \Sigma^\omega \mid \exists i \in \{1, \dots, n\}\ \exists p \in \mathsf{path}_\mathcal{A}(\alpha) \colon \mathsf{inf}(p) \cap F_i \neq \emptyset \wedge \mathsf{inf}(p) \cap G_i = \emptyset\}.$$

Show that every language recognised by a Rabin automaton is also recognised by a Büchi automaton by constructing for a given Rabin automaton $\mathcal{A}$, an S1S-formula $\phi_\mathcal{A}$ defining the language $L_\omega(\mathcal{A})$.