Exercise 53
Let $\Sigma = \Sigma_0 \cup \Sigma_1 \cup \Sigma_2$ be an alphabet with arity function, where $\Sigma_0 = \{x, y, z\}$, $\Sigma_1 = \{\neg\}$, and $\Sigma_2 = \{\land, \lor\}$. Define tree automata (either LR or RL) recognising the tree languages consisting of the following trees:

a) trees containing the symbol $\lor$ exactly once;
b) trees containing the symbol $\neg$ at least once on every path of the tree; and
c) trees describing satisfiable propositional formulae.

Exercise 54
Let $\mathcal{A} = (Q, \Sigma, I, \Delta, F)$ be the non-deterministic LR-tree automaton given by:

- $Q = \{0, 1\}$;
- $\Sigma = \{f, x, y\}$ with $\nu(f) = 2$ and $\nu(x) = \nu(y) = 0$;
- $I(x) = \{0, 1\}$ and $I(y) = \{0\}$;
- $\Delta_f(0, 0) = \{0\}$, $\Delta_f(0, 1) = \{1, 0\}$, $\Delta_f(1, 0) = \{1, 0\}$, and $\Delta_f(1, 1) = \{1\}$; and
- $F = \{1\}$.

Do the following:

a) Adapt the standard powerset construction from finite automata on words to LR-tree automata and use it to construct a deterministic LR-tree automaton $\mathcal{A}'$ such that $L(\mathcal{A}) = L(\mathcal{A}')$.

b) Try to apply a similar construction to the RL-tree automaton from Example 6.10 from the lecture. Explain why this method fails for RL-tree automata.

Exercise 55
Example 6.10 from the lecture shows that deterministic RL-tree automata recognise a smaller class of languages than non-deterministic ones. We call an RL-tree automaton $\mathcal{A} = (Q, \Sigma, I, \Delta, F)$ quasi-deterministic if

- $\Delta$ is a deterministic transition assignment, and
- $I \subseteq Q$ is a set of initial states.
Prove or refute:

a) If $L \subseteq T_\Sigma$ is a finite tree language, then there exists a quasi-deterministic tree automaton recognising $L$.

b) If $L \subseteq T_\Sigma$ is a recognisable tree language, then there exists a quasi-deterministic tree automaton recognising $L$.

Exercise 56
Devise a quadratic-time algorithm that decides the emptiness problem for LR-tree automata.

Exercise 57
Let $A = (Q, \Sigma, I, \Delta, F)$ be an RL-tree automaton given by:

- $Q = \{1, \ldots, 4\}$;
- $\Sigma = \{g, n, a, b\}$ with $\nu(g) = 2$, $\nu(n) = 1$, and $\nu(a) = \nu(b) = 0$;
- $I = \{1\}$;
- $\Delta_g(1) = \{(1,1), (1,2), (3,4), (4,1)\}$, $\Delta_g(2) = \emptyset$, $\Delta_g(3) = Q \times Q$, $\Delta_g(4) = \{(1,2), (1,4), (2,4), (2,2)\}$;
- $\Delta_n(1) = \{1\}$, $\Delta_n(2) = \{3\}$, $\Delta_n(3) = \{1, 2\}$, $\Delta_n(4) = \{1, 3\}$; and
- $F(a) = \{2\}$, $F(b) = \{2, 3\}$.

Decide whether $L(A) = \emptyset$ or not.