Automata and Logic

Exercise Sheet 12
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Exercise 58
Let $\Sigma = \{a, b\}$ be an alphabet with two binary symbols and

$$L := \{ t \in T_{\Sigma}^\omega \mid \text{there is a path in } t \text{ containing the symbol } a \text{ only finitely often}\}.$$

a) Is $L$ Rabin-recognisable?
b) Is $L$ Büchi-recognisable?

Exercise 59
Let $\Sigma$ be an alphabet (with arity function) containing at least two binary symbols $f$ and $g$.

Prove or refute that the $\omega$-tree language $L = \{ f(t, t) \mid t \in T_{\Sigma}^\omega \}$ is Büchi-recognisable.

Exercise 60
Let $\Sigma$ be an alphabet of binary symbols.

• For each $\omega$-tree $t \in T_{\Sigma}^\omega$, we define a language of $\omega$-words $\text{path}(t) \subseteq \Sigma^\omega$ as follows:

$$\text{path}(t) := \{ \alpha \in \Sigma^\omega \mid \text{there is a path } i_0i_1i_2 \ldots \text{ in } t \text{ such that } t(i_0 \ldots i_n) = \alpha(n) \text{ for all } n \in \mathbb{N}\}.$$

For $B \subseteq T_{\Sigma}^\omega$, let $\text{path}(B) := \bigcup_{t \in B} \text{path}(t)$.

• For a language $L \subseteq \Sigma^\omega$ of $\omega$-words, let $\text{tree}(L) := \{ t \in T_{\Sigma}^\omega \mid \text{path}(t) \subseteq L\}$.

Prove or refute:

a) $\text{tree}(\text{path}(B)) = B$
b) $\text{path}(\text{tree}(L)) = L$
c) If $B$ is Büchi-recognisable, then $\text{path}(B)$ is also Büchi-recognisable.
d) If $\text{path}(B)$ is Büchi-recognisable, then $B$ is also Büchi-recognisable.
e) If $L$ is Büchi recognisable, then $\text{tree}(L)$ is also Büchi-recognisable.