Automata and Logic

Exercise Sheet 12

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Exercise 58

Let $\Sigma = \{a, b\}$ be an alphabet with two binary symbols and

$$L := \{t \in \mathbf{T}_\Sigma^\omega \mid \text{there is a path in } t \text{ containing the symbol } a \text{ only finitely often}\}.$$

a) Is L Rabin-recognisable?

b) Is L Büchi-recognisable?

Exercise 59

Let Σ be an alphabet (with arity function) containing at least two binary symbols f and g. Prove or refute that the ω-tree language $L = \{f(t, t) \mid t \in \mathbf{T}_\Sigma^\omega\}$ is Büchi-recognisable.

Exercise 60

Let Σ be an alphabet of binary symbols.

- For each ω-tree $t \in \mathbf{T}_\Sigma^\omega$, we define a language of ω-words $\mathsf{path}(t) \subseteq \Sigma^\omega$ as follows:

  $$\mathsf{path}(t) := \{\alpha \in \Sigma^\omega \mid \text{there is a path } i_0 i_1 i_2 \ldots \text{ in } t \text{ such that } t(i_0 \ldots i_n) = \alpha(n) \text{ for all } n \in \mathbb{N}\}.$$

  For $B \subseteq \mathbf{T}_\Sigma^\omega$, let $\mathsf{path}(B) := \bigcup_{t \in B} \mathsf{path}(t)$.
- For a language $L \subseteq \Sigma^\omega$ of ω-words, let $\mathsf{tree}(L) := \{t \in \mathbf{T}_\Sigma^\omega \mid \mathsf{path}(t) \subseteq L\}$.

Prove or refute:

a) $\mathsf{tree}(\mathsf{path}(B)) = B$

b) $\mathsf{path}(\mathsf{tree}(L)) = L$

c) If B is Büchi-recognisable, then $\mathsf{path}(B)$ is also Büchi-recognisable.

d) If $\mathsf{path}(B)$ is Büchi-recognisable, then B is also Büchi-recognisable.

e) If L is Büchi recognisable, then $\mathsf{tree}(L)$ is also Büchi-recognisable.