TECHNISCHE UNIVERSITÄT DRESDEN

Faculty of Computer Science Institute of Theoretical Computer Science, Chair of Automata Theory

Introduction to Complexity Theory

Exercise Sheet 2

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Exercise 4

Determine which of the following statements are correct. Explain your decision.

a) $2n \in \mathcal{O}(n)$, $2n \in o(n)$, $1 \in o(n)$

b) $n \in \mathcal{O}(n \cdot \log n)$, $n \cdot \log n \in o(n^2)$

c) $2^{2^n} \in \mathcal{O}(2^n)$, $2^{2n} \in o(2^n)$

d) $4^n \in \mathcal{O}(2^n)$, $4^n \in 2^{\mathcal{O}(n)}$

e) $f(n) \in o(g(n))$ implies $f(n) \in \mathcal{O}(g(n))$ and $g(n) \notin \mathcal{O}(f(n))$

f) Show that the following two functions are a counterexample against the converse of e):

- $f(n) = 0.5 \cdot n$;
- $g(n) = \begin{cases} 2^n & \text{if } n \text{ is even} \\ n & \text{if } n \text{ is odd} \end{cases}$

Exercise 5

Let $f, g \colon \mathbb{N} \to \mathbb{N}$. We write $f \in \Theta(g)$ if $f \in \mathcal{O}(g)$ and $g \in \mathcal{O}(f)$. Show that if $p(n)$ is a polynomial of degree d, then $p(n) \in \Theta(n^d)$.

Exercise 6

Devise a non-deterministic Turing machine that accepts the following language L over $\Sigma := \{a, b\}$. For each $w_1 w_2 \dots w_n \in L$, there is a position i, $1 \leq i \leq n-2$, such that $w_i = a$ and $w_{i+2} \neq b$.

Exercise 7

Finish the proof of Thm. 2.4 from the lecture by showing that

$$(d^{T(n)+1} - 1) \cdot c \cdot T(n) \in 2^{\mathcal{O}(T(n))}.$$

Exercise 8

Show that

a) the functions $f_1(n) = 2n$ and $f_2(n) = n^2$ are time-constructible; and

b) the functions $g_1(n) = 2^n$ and $g_2(n) = n!$ is space-constructible.