Introduction to Complexity Theory

Exercise Sheet 4

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Summer Semester 2012

Exercise 13

Prove the following. It is allowed to use theorems from the lecture:

a) $\mathsf{DTime}(2^n) = \mathsf{DTime}(2^{n+1})$

b) $\mathsf{DTime}(2^n) \subsetneq \mathsf{DTime}(2^{3n})$

Exercise 14

In the lecture, it was explained that non-deterministic transitions of a Turing machine can be thought of as "guessing". For example, a word $u \in \Sigma^m$ can be guessed by m consecutive transitions, each one non-deterministically producing a symbol from Σ on the tape.

Consider NTMs N that are $\mathcal{O}(n)$-time bounded. Can such NTMs perform the following when started on an input of length n?

a) Guess a natural number between 0 and n.

b) Guess a natural number between 0 and 2^n.

c) Guess a word from $\{a, b\}^*$ of length 2^n.

d) Guess a rational number between 0 and n.

e) Guess a word from $\mathbb{N}^*$ of length n.

Exercise 15

Prove that the Hamiltonian Path Problem is in NP.

Exercise 16

An undirected graph is k-colourable iff you can colour nodes with k colours such that all adjacent nodes have different colours. Prove that 2-colourability is in P.