



Introduction to Complexity Theory

Exercise Sheet 5

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Summer Semester 2012

Exercise 17

Prove that the following problem can be solved in polynomial time: Given a Boolean formula ϕ using variables p_1, \dots, p_k and a truth assignment $w \in \{0, 1\}^k$, check whether w is a model of ϕ .

Exercise 18

Show that the following problem is in PSPACE. Given a non-deterministic finite automaton \mathcal{A} over an alphabet Σ , is $L(\mathcal{A}) = \Sigma^*$.

Hint: Prove that, if $L(\mathcal{A}) \neq \Sigma^*$ and \mathcal{A} has n states, then there exists a word $w \in \Sigma^*$ of length at most 2^n such that $w \notin L(\mathcal{A})$. Then, use this fact to give a non-deterministic algorithm whose space consumption is polynomially bounded. Finally, apply Savitch's Theorem.

Exercise 19

A *quantified Boolean formula* is of the form $\phi = Q_1 p_1 \dots Q_n p_n \cdot \psi$ where $Q_1, \dots, Q_n \in \{\forall, \exists\}$, p_1, \dots, p_n are propositional variables, and ψ is propositional formula containing only the variables p_1, \dots, p_n .

Such a formula is *valid* iff the following is satisfied:

- For $n = 0$, the formula ϕ does not contain variables, and thus is a Boolean combination of 0 and 1. It is valid iff this Boolean combination of 0 and 1 evaluates to 1.
- For $n > 0$, we consider:

$$\phi_0 := Q_2 p_2 \dots Q_n p_n \cdot \psi[p_1 := 0], \text{ and}$$

$$\phi_1 := Q_2 p_2 \dots Q_n p_n \cdot \psi[p_1 := 1].$$

If $Q_1 = \exists$, then ϕ is valid iff one of ϕ_0 and ϕ_1 is valid.

If $Q_1 = \forall$, then ϕ is valid iff both ϕ_0 and ϕ_1 are valid.

QBF denotes the set of valid quantified Boolean formulae. Prove that $\text{QBF} \in \text{PSPACE}$.