TECHNISCHE UNIVERSITÄT DRESDEN

Faculty of Computer Science Institute of Theoretical Computer Science, Chair of Automata Theory

Introduction to Complexity Theory

Exercise Sheet 6

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Exercise 20

For each of the following statements, say whether it is true or false. Justify your answers.

a) 3SAT $\in$ PSPACE;

b) SAT $\leq_p$ CLIQUE;

c) CLIQUE $\leq_p$ SAT;

d) LOGSPACE $\neq$ EXPTIME; and

e) if $L \leq_p L'$ and L' is NP-hard, then L is NP-hard.

Exercise 21

The subset-sum problem is defined as follows.

$$\text{SUBSET-SUM} := \{(S,t) \mid S = \{x_1,\dots,x_k\} \text{ and there is a } \{y_1,\dots,y_\ell\} \subseteq S \text{ such that } \sum_{i=1}^{\ell} y_i = t\}.$$

Prove that SUBSET-SUM is NP-complete.

Exercise 22

HALF-CLIQUE is the problem to decide, given a graph G with $n \geq 1$ nodes, whether G contains a clique of size $\lceil n/2 \rceil$. Prove that HALF-CLIQUE is NP-complete.

Exercise 23

Prove Lemma 4.14 from the lecture: Let $\mathcal{C}$ be a complexity class closed under polynomial-time reductions. If L is $\mathcal{C}$-hard and $L \leq_p L'$, then L' is $\mathcal{C}$-hard.

Exercise 24

Complete the proof of Thm. 4.13 from the lecture by showing that M accepts w iff $\phi_w := \phi_{\text{ini}} \wedge \phi_{\text{move}} \wedge \phi_{\text{keep}} \wedge \phi_{\text{acc}} \wedge \phi_{\text{aux}}$ is satisfiable.