Formal Concept Analysis and Logic

Exercise Sheet 2

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Summer Semester 2012

Exercise 7

It remains to prove the "only if"-direction. Let $A \to B$ follow from $\mathcal{L}$.

We assume that there is a context (G, M, I) in which all implications from $\mathcal{L}$ hold but $A \to B$ does not. Since $A \to B$ does not hold it follows that $A' \not\subseteq B'$, or equivalently according to the properties of Galois connections $B \not\subseteq A''$. Since $A \subseteq A''$, this means that A'' does not respect $A \to B$.

We prove that A'' respects all implications from $\mathcal{L}$: Let $C \to D$ be an implication from $\mathcal{L}$ with $C \subseteq A''$. From extensivity and idempotency of $\cdot''$ we get

$$C'' \subseteq A'''' = A''.$$

$C \to D$ holds in (G, M, I), which yields $C' \subseteq D'$ or equivalently

$$D \subseteq C''.$$

We have thus shown that $D \subseteq A''$ and consequently A'' respects $C \to D$. Therefore A'' must respect all implications from $\mathcal{L}$ which contradicts the assumption that $A \to B$ follows from $\mathcal{L}$.