TECHNISCHE UNIVERSITÄT DRESDEN

Faculty of Computer Science Institute for Theoretical Computer Science, Chair for Automata Theory

Formal Concept Analysis and Logic

Exercise Sheet 4 (Solutions)

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Exercise 13

We abbreviate each attribute by its first letter. As the order on the attributes we choose $e < i < a < o < r$.

We explain the first steps in detail. The Next-Closure algorithm for computing all pseudo-closed sets is always initialized by $P_0 = \emptyset$ and $\mathcal{L}_0 = \emptyset$. We need to check whether P_0 is an intent or pseudo-intent by computing the closure. We obtain $P_0'' = \emptyset = P_0$, i.e. it is an intent. In this case nothing is added to the base: $\mathcal{L}_1 = \mathcal{L}_0 = \emptyset$.

Next, we need to find $x_{\max}$. We start with r (the largest attribute) and check whether it satisfies $r <_r P_0 \oplus_{\mathcal{L}_1} r$. We obtain

$$P_0 \oplus_{\mathcal{L}_1} r = \mathcal{L}_1(\{r\}) = \{r\},$$

because $\mathcal{L}_1$ is empty. Hence $x_{\max} = r$ and therefore $P_1 = \{r\}$.

Now, the second iteration starts, ie. we need to check again whether P_1 is closed, etc. In the following iterations we obtain

- $P_1 = \{r\} = P_1''$, $\mathcal{L}_2 = \mathcal{L}_1 = \emptyset$
- $P_2 = \{o\} = P_2''$, $\mathcal{L}_3 = \mathcal{L}_2 = \emptyset$
- $P_3 = \{o, r\} \neq P_3'' = \{e, i, a, o, r\}$, $\mathcal{L}_4 = \{\{o, r\} \to \{e, i, a, o, r\}\}$ (since P_3 is not closed it is a pseudo-intent and we need to add the corresponding implication)
- $P_4 = \{a\} = P_4''$, $\mathcal{L}_5 = \mathcal{L}_4$
- $P_5 = \{a, r\} \neq P_5'' = \{e, i, a, o, r\}$, $\mathcal{L}_6 = \{\{o, r\} \to \{e, i, a, o, r\}, \{a, r\} \to \{e, i, a, o, r\}\}$
- $P_6 = \{a, o\} \neq P_6'' = \{e, i, a, o, r\}$,
  $\mathcal{L}_7 = \{\{o, r\} \to \{e, i, a, o, r\}, \{a, r\} \to \{e, i, a, o, r\}, \{a, o\} \to \{e, i, a, o, r\}\}$
- $P_7 = \{i\} = P_7''$, $\mathcal{L}_8 = \mathcal{L}_7$
- $P_8 = \{i, r\} = P_8''$, $\mathcal{L}_9 = \mathcal{L}_8$
- $P_9 = \{i, o\} = P_9''$, $\mathcal{L}_{10} = \mathcal{L}_9$
- $P_{10} = \{i, a\} = P_{10}''$, $\mathcal{L}_{11} = \mathcal{L}_{10}$

- $P_{11} = \{e\} \neq P_{11}''\{e, i, a\}$, $\mathcal{L}_{12} = \{\{o, r\} \to \{e, i, a, o, r\}, \{a, r\} \to \{e, i, a, o, r\}, \{a, o\} \to \{e, i, a, o, r\}, \{e\} \to \{e, i, a\}\}$
- $P_{12} = \{e, i, a\} = P_{12}''$, $\mathcal{L}_{13} = \mathcal{L}_{12}$.

Hence, the Duquenne-Guigues Base is

$$\begin{aligned}\mathcal{L} = \{\{\{o, r\} &\to \{e, i, a, o, r\},\\ \{a, r\} &\to \{e, i, a, o, r\}\\ \{a, o\} &\to \{e, i, a, o, r\},\\ \{e\} &\to \{e, i, a\}\}.\end{aligned}$$

Exercise 14

Observe that

- $\sqcap$ can be expressed using $\neg$ and $\sqcup$ using de-Morgans laws (and vice versa)
- $\exists$ can be expressed using $\neg$ and $\forall$ using $\exists r.C \equiv \neg\forall r.\neg C$ (and vice versa)

Therefore, there are 4 minimal fragments:

- $\neg$, $\sqcap$, $\forall$
- $\neg$, $\sqcap$, $\exists$
- $\neg$, $\sqcup$, $\forall$
- $\neg$, $\sqcup$, $\exists$

Exercise 15

We obtain

a) $\{d, e, f\}$
b) $\{g\}$
c) $\{d, e, f\}$
d) $\{g\}$
e) $\{d, f, g\}$
f) $\{e\}$