

Formal Concept Analysis and Logic

Exercise Sheet 5

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Exercise 18

Initially, the set of objects is empty. We choose the order $C < D < E$ on the attributes.

1st Iteration

$$P_0 = \emptyset, P_0'' = \{C, D, E\}$$

Question to reasoner: Does $\text{lcs} \emptyset \sqsupseteq_{\mathcal{T}} \text{lcs}\{C, D, E\}$ hold? (Equivalently: $\perp \sqsupseteq_{\mathcal{T}} \exists r.T?$)

Answer: No.

Counterexample: \perp

	C	D	E
\perp			

New $P_0'' = \emptyset = P_0$, i.e. P_0 is a intent.

2nd Iteration

$$P_1 = \{E\}, P_1'' = \{C, D, E\}$$

Question to reasoner: Does $\text{lcs}\{E\} \sqsupseteq_{\mathcal{T}} \text{lcs}\{C, D, E\}$ hold? (Equivalently: $E \sqsupseteq_{\mathcal{T}} \exists r.T?$)

Answer: No.

Counterexample: A

	C	D	E
\perp			
A		×	×

New $P_1'' = \{D, E\}$.

Question to reasoner: Does $\text{lcs}\{E\} \sqsupseteq_{\mathcal{T}} \text{lcs}\{D, E\}$ hold? (Equivalently: $E \sqsupseteq_{\mathcal{T}} A \sqcap \exists r.T?$)

Answer: No.

Counterexample: $\exists r.B$

	C	D	E
\perp			
A		\times	\times
$\exists r.B$			\times

New $P_1'' = \{E\}$. P_1 is an intent.

3rd Iteration

$P_2 = \{D\}$, $P_2'' = \{D, E\}$

Question to reasoner: Does $\text{lcs}\{D\} \sqsubseteq_{\mathcal{T}} \text{lcs}\{D, E\}$ hold? (Equivalently: $D \sqsubseteq_{\mathcal{T}} A \sqcap \exists r.T$?)

Answer: No.

Counterexample: B

	C	D	E
\perp			
A		\times	\times
$\exists r.B$			\times
B	\times	\times	

New $P_2'' = \{D\}$. P_2 is an intent.

4th Iteration

$P_3 = \{D, E\}$, $P_3'' = \{D, E\}$. P_3 is an intent

5th Iteration

$P_4 = \{C\}$, $P_4'' = \{C, D\}$.

Question to reasoner: Does $\text{lcs}\{C\} \sqsubseteq_{\mathcal{T}} \text{lcs}\{C, D\}$ hold? (Equivalently: $C \sqsubseteq_{\mathcal{T}} B \sqcap \exists r.T$?)

Answer: No.

Counterexample: $\exists r.A$

	C	D	E
\perp			
A		\times	\times
$\exists r.B$			\times
B	\times	\times	
$\exists r.A$	\times		

New $P_4'' = \{C\}$. P_4 is an intent.

6th Iteration

$$P_5 = \{C, E\}, P'_5 = \{C, D, E\}.$$

Question to reasoner: Does $\text{lcs}\{C, E\} \sqsupseteq_{\mathcal{T}} \text{lcs}\{C, D, E\}$ hold? (Equivalently: $\exists r.T \sqsupseteq_{\mathcal{T}} \exists r.T$?)

Answer: Yes.

$$\mathcal{L}_5 = \{CE \rightarrow CDE\}$$

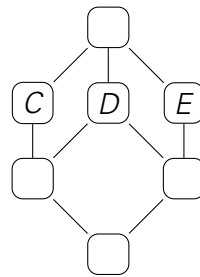
7th Iteration

$$P_6 = \{C, D\}, P'_6 = \{C, D\}. P_6 \text{ is intent.}$$

8th Iteration

$$P_7 = \{C, D, E\}, P'_7 = \{C, D, E\}. P_7 \text{ is intent.}$$

We obtain the following concept lattice (attribute labels only).



The inverse of this lattice is the subsumption hierarchy of all least common subsumers.

