Exercise 18
Initially, the set of objects is empty. We choose the order $C < D < E$ on the attributes.

1st Iteration

$P_0 = \emptyset, \quad P''_0 = \{C, D, E\}$

**Question to reasoner:** Does $\text{lcs} \emptyset \sqsupseteq_T \text{lcs} \{C, D, E\}$ hold? (Equivalently: $\bot \sqsupseteq_T \exists r. \top$?)

**Answer:** No.

**Counterexample:** $\bot$

\[ \begin{array}{ccc}
\bot & C & D & E \\
\end{array} \]

New $P''_0 = \emptyset = P_0$, i.e. $P_0$ is a intent.

2nd Iteration

$P_1 = \{E\}, \quad P''_1 = \{C, D, E\}$

**Question to reasoner:** Does $\text{lcs} \{E\} \sqsupseteq_T \text{lcs} \{C, D, E\}$ hold? (Equivalently: $E \sqsupseteq_T \exists r. \top$?)

**Answer:** No.

**Counterexample:** $A$

\[ \begin{array}{ccc}
A & C & D & E \\
\bot & & & \\
\end{array} \]

New $P''_1 = \{D, E\}$.

**Question to reasoner:** Does $\text{lcs} \{E\} \sqsupseteq_T \text{lcs} \{D, E\}$ hold? (Equivalently: $E \sqsupseteq_T A \cap \exists r. \top$?)

**Answer:** No.
Counterexample: \( \exists r. B \)

\[
\begin{array}{ccc}
C & D & E \\
\bot & & \\
A & \times & \times \\
\exists r. B & \\
\end{array}
\]

New \( P_1'' = \{E\} \). \( P_1 \) is an intent.

3rd Iteration

\( P_2 = \{D\}, \ P_2'' = \{D, E\} \)

**Question to reasoner:** Does \( \text{lcs}\{D\} \supseteq \text{lcs}\{D, E\} \) hold? (Equivalently: \( D \supseteq A \cap \exists r. T? \))

**Answer:** No.

**Counterexample:** \( B \)

\[
\begin{array}{ccc}
C & D & E \\
\bot & & \\
A & \times & \times \\
\exists r. B & \\
B & \times & \times \\
\end{array}
\]

New \( P_2'' = \{D\} \). \( P_2 \) is an intent.

4th Iteration

\( P_3 = \{D, E\}, \ P_3'' = \{D, E\} \). \( P_3 \) is an intent

5th Iteration

\( P_4 = \{C\}, \ P_4'' = \{C, D\} \).

**Question to reasoner:** Does \( \text{lcs}\{C\} \supseteq \text{lcs}\{C, D\} \) hold? (Equivalently: \( C \supseteq B \cap \exists r. T? \))

**Answer:** No.

**Counterexample:** \( \exists r. A \)

\[
\begin{array}{ccc}
C & D & E \\
\bot & & \\
A & \times & \times \\
\exists r. B & \\
B & \times & \times \\
\exists r. A & \\
\end{array}
\]

New \( P_4'' = \{C\} \). \( P_4 \) is an intent.
6th Iteration

\( P_5 = \{C, E\}, \ P'_5 = \{C, D, E\}. \)

**Question to reasoner:** Does \( \text{lcs}\{C, E\} \models_T \text{lcs}\{C, D, E\} \) hold? (Equivalently: \( \exists r. T \models_F \exists r. T \)?)

**Answer:** Yes.

\( L_5 = \{CE \rightarrow CDE\} \)

7th Iteration

\( P_6 = \{C, D\}, \ P'_6 = \{C, D\}. \ P_6 \) is intent.

8th Iteration

\( P_7 = \{C, D, E\}, \ P'_7 = \{C, D, E\}. \ P_7 \) is intent.

We obtain the following concept lattice (attribute labels only).

![Concept Lattice](image)

The inverse of this lattice is the subsumption hierarchy of all least common subsumers.

![Subsumption Hierarchy](image)