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Faculty of Computer Science Institute for Theoretical Computer Science, Chair for Automata Theory

Formal Concept Analysis and Logic

Exercise Sheet 1

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Summer Semester 2012

Exercise 1

Let $\mathfrak{B}$ be the set of all propositions formulas in the variables $x_1, \ldots, x_n$. Let $\mathfrak{V}$ be the set of all truth-value-assignments for these variables.

a) For each subset $T \subseteq \mathfrak{B}$ define

$$c(T) = \{f \in \mathfrak{B} : T \models f\},$$

the set of all formulas that are entailed by T. Prove that c is a closure operator on $(2^{\mathfrak{B}}, \subseteq)$.

b) For each subset $T \subseteq \mathfrak{B}$ and each subset $S \subseteq \mathfrak{V}$ define

$$F(T) = \{v \in \mathfrak{V} \mid v \text{ evaluates all } f \in T \text{ to true}\},$$
$$G(S) = \{f \in \mathfrak{B} \mid \text{all } v \in S \text{ evaluate } f \text{ to true}\}.$$

Show that (F, G) forms a Galois-connection.

Exercise 2

Complete the prove of Lemma 2.8 by showing that the pair (f, g) is a Galois-connection between $(A, \leq)$ and $(B, \preceq)$ if

- $g \circ f$ and $f \circ g$ are extensive, and
- f and g are antitonic.

Exercise 3

In the lecture we have shown that the composition of the two derivation operators is a closure operator. In a formal context sets of attributes $B \subseteq M$ are closed if they satisfy $B = B''$.

Let a formal context, about the ruling parties in several states of Germany, be given by the following cross table:

	CDU/CSU	SPD	Grüne	Linke	FDP
Baden-Württemberg		×	×		
Bayern	×				×
Berlin	×	×			
Brandenburg		×		×	
Bremen		×	×		
Hamburg		×			
Mecklenburg-Vorpommern	×	×			
Niedersachsen	×				×
NRW		×	×		
Rheinland-Pfalz		×	×		
Saarland	×				
Sachsen	×				×
Sachsen-Anhalt	×	×			
Schleswig-Holstein	×				×
Thüringen	×	×			

a) Can there be a closed set of attributes with 3 elements?

b) What are the closed sets of attributes with 2 elements?

Exercise 4

Let $n \in \mathbb{N}$ be a given natural number. Can you find a formal context (G, M, I) such that

- G has n elements, and
- there are 2^n closed sets of objects.