TECHNISCHE UNIVERSITÄT DRESDEN

**Faculty of Computer Science** Institute for Theoretical Computer Science, Chair for Automata Theory

# Formal Concept Analysis and Logic

**Exercise Sheet 3**

Dr. Felix Distel

Summer Semester 2012

## Exercise 9

Let $n \geq 3$ be a natural number.

- Present a formal context such that there are $n$ pseudo-intents, but at least $2^{n-1}$ intents.
- Show that contexts of the following form have $2^n$ pseudo-intents.

| | $m_0$ | $m_1 \quad \dots \quad m_n$ | $m_{n+1} \quad \dots \quad m_{2n}$ |
|---|---|---|---|
| $g_1$ | | | |
| $\vdots$ | | $\neq$ | $\neq$ |
| $g_n$ | | | |
| $g_{n+1}$ | $\times$ | | |
| $\vdots$ | $\vdots$ | $\neq$ | |
| $g_{3n}$ | $\times$ | | |

Here $\neq$ stands for the contranominal scale (where there are crosses everywhere except on the main diagonal).

## Exercise 10

Complete the proof of Lemma 2.35. Show that the pseudo-closure operator $\mathcal{D}G^*_{(G,M,I)}$ is a closure operator, i.e. it is extensive, monotone and idempotent.

Show that all intents and pseudo-intents are pseudo-closed.

## Exercise 11

Prove Lemma 2.37 which states that for a pseudo-closed set $P$ the lectically next pseudo-closed set $\bar{P}$ is of the form

$$P \oplus_{\mathcal{L}} x,$$

where $\mathcal{L} = \{Q \to Q'' \mid Q < P\}$, and $x$ is maximal with the property that

$$P <_x P \oplus_{\mathcal{L}} x.$$

**Exercise 12**

**(optional exercise)**

A set $Q \subseteq M$ is called a *quasi-intent* iff every $R \subseteq Q$ satisfies $R'' \subseteq Q$ or $R'' = Q''$.

Prove that a set $P \subseteq M$ is a pseudo-intent iff

- $P \neq P''$,
- $P$ is a quasi-intent, and
- for all quasi-intents $Q$, $Q \subsetneq P$ implies $Q'' \subsetneq P$.