Formal Concept Analysis and Logic

Exercise Sheet 4
Dr. Felix Distel
Summer Semester 2012

Exercise 13

Use Next-Closure to compute the Duquenne-Guigues Base of the following context.

<table>
<thead>
<tr>
<th></th>
<th>equilateral</th>
<th>isosceles</th>
<th>acute-angled</th>
<th>obtuse angled</th>
<th>right-angled</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_1$</td>
<td>×</td>
<td></td>
<td></td>
<td></td>
<td>×</td>
</tr>
<tr>
<td>$D_2$</td>
<td>×</td>
<td></td>
<td></td>
<td>×</td>
<td></td>
</tr>
<tr>
<td>$D_3$</td>
<td>×</td>
<td>×</td>
<td></td>
<td>×</td>
<td></td>
</tr>
<tr>
<td>$D_4$</td>
<td>×</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$D_5$</td>
<td>×</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$D_6$</td>
<td>×</td>
<td>×</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$D_7$</td>
<td>×</td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

Exercise 14

Recall that the description logic $\mathcal{ALC}$ is equipped with the concept constructors negation ($\neg$), conjunction ($\land$), disjunction ($\lor$), existential restriction ($\exists r.C$), and universal restriction ($\forall r.C$). Each subset of this set of constructors gives rise to a fragment of $\mathcal{ALC}$.

Identify all minimal fragments that are equivalent to $\mathcal{ALC}$ in the sense that for every $\mathcal{ALC}$-concept, there is an equivalent concept in the fragment.

Two concepts are equivalent iff the concepts have the same extension in every interpretation.
Exercise 15

Consider the (graphical representation of the) interpretation $\mathcal{I}$ with $\Delta^\mathcal{I} = \{d, e, f, g\}$:

For each of the following $\mathcal{ALC}$-concepts $C$, list all elements $x$ of $\Delta^\mathcal{I}$ such that $x \in C^\mathcal{I}$:

a) $A \sqcup B$

b) $\exists s. \neg A$

c) $\forall s. A$

d) $\exists s. \exists s. \exists s. A$

e) $\neg \exists r. (\neg A \sqcap \neg B)$

f) $\exists s. (A \sqcap \forall s. \neg B) \sqcap \neg \forall r. \exists r. (A \sqcup \neg A)$