Formal Concept Analysis and Logic

Exercise Sheet 5

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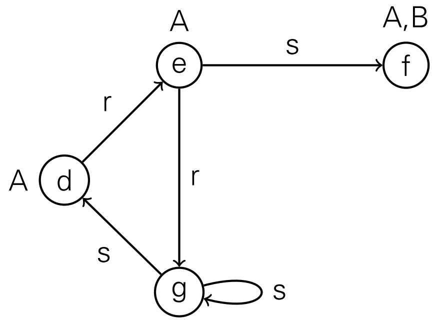
Summer Semester 2012

Exercise 16

Consider the ABox

$$\begin{aligned}\mathcal{A} = \{&A(d), A(e), A(f),\\ &B(f),\\ &r(d,e), r(e,g),\\ &s(e,f), s(g,d), s(g,g)\}\end{aligned}$$

which has the graphical representation:



For each of the following $\mathcal{ALC}$-concepts C, list all individual names $a \in \mathcal{N}_C$ such that $C(a)$ is entailed by $\mathcal{T}$, $\mathcal{A}$.

a) $A \sqcup B$

b) $\exists s.\neg A$

c) $\forall s.A$

d) $\exists s.\exists s.\exists s.\exists s.A$

e) $\neg\exists r.(\neg A \sqcap \neg B)$

f) $\exists s.(A \sqcap \forall s.\neg B) \sqcap \neg\forall r.\exists r.(A \sqcup \neg A)$

Compare your results to the results from Exercise 15. Are they different? If yes, why are they different?

Exercise 17

Most standard reasoning problems in DL can be reduced to consistency.

a) Show that $C \sqsubseteq_{\mathcal{T}} D$ iff $\mathcal{T} \cup \{C \sqcap \neg D(a)\}$ is inconsistent.

b) Satisfiability is the following reasoning problem: Given an ontology $\mathcal{O}$ and a concept C decide whether there is a model $\mathcal{I}$ such that $C^{\mathcal{I}}$ is not empty. How can this problem be reduced to consistency?

Exercise 18

Consider the TBox

$$\begin{aligned}\mathcal{T} = \{&C \equiv B \sqcap \exists r.A,\\ &D \equiv A \sqcap B \sqcap \exists r.\top,\\ &E \equiv A \sqcap \exists r.B\}.\end{aligned}$$

a) Show that $B \sqcap \exists r.\top$ is the least common subsumer of C and D.

b) What is the least common subsumer of D and E? What is the least common subsumer of C and E?

c) Use Lemma 3.5 and Attribute Exploration to obtain the subsumption hierarchy of all least common subsumers of subsets of $\{C, D, E\}$.