TECHNISCHE UNIVERSITÄT DRESDEN

Faculty of Computer Science Institute of Theoretical Computer Science, Chair of Automata Theory

Automata and Logic

Exercise Sheet 2

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Exercise 6

Prove that the language $L := \{a^n b^n \mid n \geq 0\}$ is *not* regular using Nerode's Theorem.

Exercise 7

Let $M := \{1, m\}$ with $1 \neq m$. Determine all operations $\circ$ such that $(M, \circ, 1)$ is a monoid.

Exercise 8

Consider the monoid $(\mathbb{Z}, +, 0)$ and the following relations on $\mathbb{Z}$, where $3 \mid z$ denotes that z is divided by 3 without remainder.

- $z_1 R_1 z_2$ iff $3 \mid (z_1 - z_2)$;
- $z_1 R_2 z_2$ iff $3 \mid z_1$ and $3 \mid z_2$, or $3 \nmid z_1$ and $3 \nmid z_2$.

For each $z \in \mathbb{Z}$, $[z]_i$ denotes the equivalence class of z w.r.t. the relation R_i. We now define the monoids $(M_i, \circ_i, 1_i)$ for $i \in \{1, 2\}$ as follows:

- $M_i := \{[z]_i \mid z \in \mathbb{Z}\}$
- $[z]_i \circ_i [z']_i := [z + z']_i$
- $1_i := [0]_i$

Prove the following:

a) R_1 and R_2 are both equivalence relations.

b) R_1 is a congruence relation, but R_2 is not.

c) $(M_1, \circ_1, 1_1)$ is well-defined, but $(M_2, \circ_2, 1_2)$ is not.

Exercise 9

Let $\mathcal{A} = (Q, \Sigma, q_0, \delta, F)$ be a deterministic finite automaton. In the lecture, we defined the relations $\sim_{\mathcal{A}}, \sim_0, \sim_1, \ldots \subseteq Q \times Q$ as follows:

- $q \sim_{\mathcal{A}} q'$ iff $L(\mathcal{A}_q) = L(\mathcal{A}_{q'})$;
- $q \sim_0 q'$ iff $\{q, q'\} \subseteq F$ or $\{q, q'\} \cap F = \emptyset$;
- $q \sim_{i+1} q'$ iff $q \sim_i q'$ and $\delta(q, a) \sim_i \delta(q', a)$ for all $a \in \Sigma$.

Prove that there exists an $n \in \mathbb{N}$ such that $\sim_n = \sim_{\mathcal{A}}$.

Exercise 10

Consider the monoids $M_i := (\{1, a, b\}, \circ_i, 1)$, for $i \in \{1, 2\}$, where $\circ_1$ is given by the following table:

$\circ_1$	1	a	b
1	1	a	b
a	a	a	b
b	b	a	b

and $x \circ_2 y := y \circ_1 x$ for all $x, y \in \{1, a, b\}$.

For each $i \in \{1, 2\}$, find a regular language $L_i \subseteq \{a, b\}^*$ such that M_i is the syntactic monoid of L_i, or prove that no such language exists.

Exercise 11

Let Σ be an alphabet and $(M, \circ, 1)$ a monoid. Prove that every function $f : \Sigma \to M$ can be uniquely extended to a (monoid) homomorphism $\Phi : \Sigma^* \to M$.