TECHNISCHE UNIVERSITÄT DRESDEN

Faculty of Computer Science Institute of Theoretical Computer Science, Chair of Automata Theory

Automata and Logic

Exercise Sheet 4

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Exercise 19

Let V be the M-variety of all commutative finite groups. Show that there exists a language $L \subseteq \{a\}^*$ such that $L \in L(V)_{\{a\}}$ but $L \notin L(V)_{\{a,b\}}$.

Exercise 20

Prove or refute the following: There is a language $L \subseteq \{a, b\}^*$ such that its syntactic semigroup S_L and its syntactic monoid M_L are isomorphic.

Exercise 21

For each of the following words over the alphabet $\{0, 1\}^k$, give a corresponding interpretation over the predicate symbols $P_1, \ldots, P_k$ as discussed in the lecture:

$$\begin{aligned} k = 2:&\quad (1,1),(1,1),(0,1),(1,0)\\ k = 3:&\quad (0,0,0),(1,0,0),(0,1,0),(0,0,1)\\ k = 3:&\quad (1,1,0),(1,0,1),(1,1,1),(1,1,0) \end{aligned}$$

Describe all interpretations that correspond to words of the language $L(((0,1)\cdot(1,0))^+) \subseteq (\{0,1\}^2)^+$.

Exercise 22

Let $\Sigma = \{a, b\}$. For each of the following regular expressions r_i, give a first-order formula ϕ_i such that $L(r_i) = L(\phi_i)$.

a) $r_1 = \Sigma^*$,

b) $r_2 = \varepsilon$,

c) $r_3 = (abb^*)^*$,

d) $r_4 = a^*b^* + b^*a^*$, and

e) $r_5 = (aaa \cdot \Sigma^*) + b^*$.

Exercise 23

Complete the proof of Lemma 2.2 from the lecture by showing that the class $(B_0)_\Sigma$ is closed under union.

Exercise 24

Let $(S, \circ)$ be a finite semigroup, $m \in S$, and $i, k, \ell \in \mathbb{N} \setminus \{0\}$ defined as in the proof of Thm. 2.4. Show that if k is minimal with the property described in the proof, then

$$(\{m^i, \dots, m^{i+k-1}\}, \circ, m^\ell)$$

is a group. Is $(\{m^i, \dots, m^{i+k-1}\}, \circ, m^\ell)$ still a group if k is not minimal?

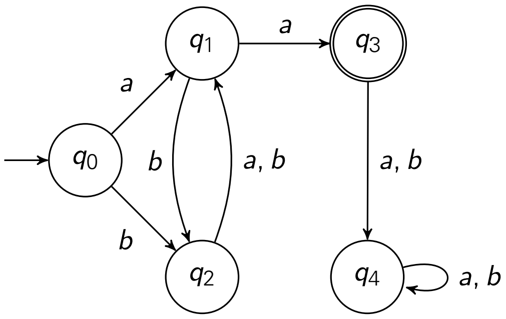
Exercise 25

Let $\Sigma := \{a, b\}$ and L_1, L_2 be the languages accepted by the automata displayed below. *Use the proof of Cor. 2.10* from the lecture to show that $L_1 \notin (B_0)_\Sigma$ and $L_2 \in (B_0)_\Sigma$.

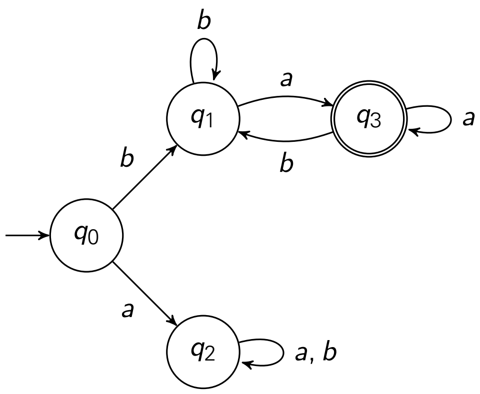
Moreover, represent L_2 as a Boolean combination of languages from the set

$$\{u\Sigma^* \mid u \in \Sigma^*\} \cup \{\Sigma^* u \mid u \in \Sigma^*\}.$$

(1)



(2)



Exercise 26

Prove or refute the following:

a) For every alphabet Σ and word $w \in \Sigma^*$, we have $\{w\} \in (B_0)_\Sigma$.

b) For every two alphabets Σ and Σ' with $\Sigma \subseteq \Sigma'$, and every language $L \subseteq \Sigma^*$, we have: if $L \in (B_0)_\Sigma$, then $L \in (B_0)_{\Sigma'}$.

c) Let $(M, \circ, 1)$ be a monoid, where 1 is the only idempotent element of M. Then $(M, \circ, 1)$ is a group.

d) Let $(S, \circ)$ be a semigroup with $e \in S$ being idempotent. Then $(eSe, \circ, e)$ is the largest submonoid of S with e as unit element.

e) Let $(S, \circ) \in \widehat{\mathbb{D}}$. If there exists an element $s \in S$ such that $(S, \circ, s)$ is a monoid, then $|S| = 1$.