TECHNISCHE UNIVERSITÄT DRESDEN

Faculty of Computer Science Institute of Theoretical Computer Science, Chair of Automata Theory

Automata and Logic

Exercise Sheet 6

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Exercise 33

Let $\Sigma = \{a\}$. Recall from the lecture that $L_{k,n}$ denotes the set of all first-order formulae (over the non-logical symbols $=$, $<$, and Q_a) containing k free variables and having quantifier depth at most n. For the following combinations of k, n, determine a *finite* set $\Gamma_{k,n}$ such that for every formula $\phi \in L_{k,n}$, there is a formula $\psi \in \Gamma_{k,n}$ with $\phi \equiv \psi$. Determine also the equivalence classes of $\equiv_{k,n}$.

a) $k = 1$, $n = 0$;

b) $k = 2$, $n = 0$;

c) $k = 0$, $n = 1$; and

d) $k = 1$, $n = 1$.

Exercise 34

Give the formulae ϕ_W for each equivalence class W of $\equiv_{2,0}$. Then, determine a finite disjunction of formulae ϕ_W for $\equiv_{2,0}$-classes, which is equivalent to the formulae:

a) true;

b) $\neg(x < y) \lor x = y$; and

c) false.

Exercise 35

Consider the Ehrenfeucht-Fraïssé games on the words

a) *ab* and *ba*; and

b) *aaabaaa* and *aabaaa*.

Determine the $k \in \{1, \ldots, 4\}$ such that Player I has a winning strategy in k moves.

Exercise 36

Consider the Ehrenfeucht-Fraïssé games on the words a^i and a^j with $i < j$.

a) Describe an optimal winning strategy for Player I, i.e. a strategy such that Player I wins with a minimal number of moves.

b) Prove that Player I has a winning strategy on a^i and a^j (with $i < j$) in m moves if $i < 2^m - 1$.