TECHNISCHE UNIVERSITÄT DRESDEN

**Faculty of Computer Science** Institute of Theoretical Computer Science, Chair of Automata Theory

# Automata and Logic

## Exercise Sheet 11

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### Exercise 55

Let $\Sigma$ be an alphabet with arity function, $x, y \in \Sigma_0$, and $U, V, W \subseteq \mathbf{T}_\Sigma$. Prove or refute:

- $U \cdot^x (V \cup W) = (U \cdot^x V) \cup (U \cdot^x W)$;
- $(U \cdot^x V) \cdot^y W = U \cdot^x (V \cdot^y W)$;
- $(U^{*,x})^{*,y} = (U^{*,y})^{*,x}$.

### Exercise 56

Example 6.10 from the lecture shows that deterministic RL-tree automata recognise a smaller class of languages than non-deterministic ones. We call an RL-tree automaton $\mathcal{A} = (Q, \Sigma, I, \Delta, F)$ *quasi-deterministic* if

- $\Delta$ is a deterministic transition assignment, and
- $I \subseteq Q$ is a *set* of initial states.

Prove or refute:

a) If $L \subseteq \mathbf{T}_\Sigma$ is a *finite* tree language, then there exists a quasi-deterministic tree automaton recognising $L$.

b) If $L \subseteq \mathbf{T}_\Sigma$ is a *recognisable* tree language, then there exists a quasi-deterministic tree automaton recognising $L$.

### Exercise 57

Devise a quadratic-time algorithm that decides the emptiness problem for LR-tree automata.

### Exercise 58

Let $\mathcal{A} = (Q, \Sigma, I, \Delta, F)$ be an RL-tree automaton given by:

- $Q = \{1, \dots, 4\}$;
- $\Sigma = \{g, n, a, b\}$ with $\nu(g) = 2$, $\nu(n) = 1$, and $\nu(a) = \nu(b) = 0$;
- $I = \{1\}$;
- $\Delta_g(1) = \{(1, 1), (1, 2), (3, 4), (4, 1)\}$, $\Delta_g(2) = \emptyset$, $\Delta_g(3) = Q \times Q$, $\Delta_g(4) = \{(1, 2), (1, 4), (2, 4), (2, 2)\}$;

- $\Delta_n(1) = \{1\}$, $\Delta_n(2) = \{3\}$, $\Delta_n(3) = \{1, 2\}$, $\Delta_n(4) = \{1, 3\}$; and
- $F(a) = \{2\}$, $F(b) = \{2, 3\}$.

Decide whether $L(\mathcal{A}) = \emptyset$ or not.

**Exercise 59**

Let $\Sigma = \{a, b\}$ be an alphabet with two binary symbols and

$$L := \{t \in \mathbf{T}^{\omega}_{\Sigma} \mid \text{there is a path in } t \text{ containing the symbol } a \text{ only finitely often}\}.$$

a) Is $L$ Rabin-recognisable?

b) Is $L$ Büchi-recognisable?

**Exercise 60**

Let $\Sigma$ be an alphabet (with arity function) containing at least two binary symbols $f$ and $g$. Prove or refute that the $\omega$-tree language $L = \{f(t, t) \mid t \in \mathbf{T}^{\omega}_{\Sigma}\}$ is Büchi-recognisable.