Automata and Logic
Exercise Sheet 11
Dr. rer. nat. Rafael Peñaloza
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Exercise 55
Let \( \Sigma \) be an alphabet with arity function, \( x, y \in \Sigma_0 \), and \( U, V, W \subseteq T_\Sigma \). Prove or refute:

- \( U \cdot^x (V \cup W) = (U \cdot^x V) \cup (U \cdot^x W) \);
- \( (U \cdot^x V)^.y W = U \cdot^x (V ^.y W) \);
- \( (U^{*x})^*.y = (U^*.y)^{*.x} \).

Exercise 56
Example 6.10 from the lecture shows that deterministic RL-tree automata recognise a smaller class of languages than non-deterministic ones. We call an RL-tree automaton \( \mathcal{A} = (Q, \Sigma, I, \Delta, F) \) quasi-deterministic if

- \( \Delta \) is a deterministic transition assignment, and
- \( I \subseteq Q \) is a set of initial states.

Prove or refute:

a) If \( L \subseteq T_\Sigma \) is a finite tree language, then there exists a quasi-deterministic tree automaton recognising \( L \).

b) If \( L \subseteq T_\Sigma \) is a recognisable tree language, then there exists a quasi-deterministic tree automaton recognising \( L \).

Exercise 57
Devise a quadratic-time algorithm that decides the emptiness problem for LR-tree automata.

Exercise 58
Let \( \mathcal{A} = (Q, \Sigma, I, \Delta, F) \) be an RL-tree automaton given by:

- \( Q = \{1, \ldots, 4\} \);
- \( \Sigma = \{g, n, a, b\} \) with \( \nu(g) = 2, \nu(n) = 1, \) and \( \nu(a) = \nu(b) = 0 \);
- \( I = \{1\} \);
- \( \Delta_g(1) = \{(1, 1), (1, 2), (3, 4), (4, 1)\}, \Delta_g(2) = \emptyset, \Delta_g(3) = Q \times Q, \)
  \( \Delta_g(4) = \{(1, 2), (1, 4), (2, 4), (2, 2)\} \);
• \( \Delta_n(1) = \{1\}, \Delta_n(2) = \{3\}, \Delta_n(3) = \{1, 2\}, \Delta_n(4) = \{1, 3\}; \) and
• \( F(a) = \{2\}, F(b) = \{2, 3\}. \)

Decide whether \( L(A) = \emptyset \) or not.

**Exercise 59**
Let \( \Sigma = \{a, b\} \) be an alphabet with two binary symbols and
\[
L := \{ t \in T_\Sigma^\omega \mid \text{there is a path in } t \text{ containing the symbol } a \text{ only finitely often} \}.
\]
a) Is \( L \) Rabin-recognisable?
b) Is \( L \) Büchi-recognisable?

**Exercise 60**
Let \( \Sigma \) be an alphabet (with arity function) containing at least two binary symbols \( f \) and \( g \).
Prove or refute that the \( \omega \)-tree language \( L = \{ f(t, t) \mid t \in T_\Sigma^\omega \} \) is Büchi-recognisable.