Exercise 1

Recall that the description logic $\mathcal{ALC}$ is equipped with the concept constructors negation ($\neg$), conjunction ($\sqcap$), disjunction ($\sqcup$), existential restriction ($\exists r.C$), and universal restriction ($\forall r.C$). Each subset of this set of constructors gives rise to a fragment of $\mathcal{ALC}$.

Identify all minimal fragments that are equivalent to $\mathcal{ALC}$ in the sense that for every $\mathcal{ALC}$-concept, there is an equivalent concept in the fragment.

Two concepts are equivalent iff the concepts have the same extension in every interpretation.

Exercise 2

Consider the (graphical representation of the) interpretation $\mathcal{I}$ with $\Delta^\mathcal{I} = \{d, e, f, g\}$:

For each of the following $\mathcal{ALC}$-concepts $C$, list all elements $x$ of $\Delta^\mathcal{I}$ such that $x \in C^\mathcal{I}$:

a) $A \sqcup B$

b) $\exists s. \neg A$

c) $\forall s. A$

d) $\exists s. \exists s. \exists s. A$

e) $\neg \exists r. (\neg A \sqcap \neg B)$

f) $\exists s. (A \sqcap \forall s. \neg B) \sqcap \neg \forall r. \exists r. (A \sqcup \neg A)$
Exercise 3
In addition to the concept assertions presented in the lecture, ABoxes are sometimes allowed to contain role assertions of the form \( r(a, b) \). An interpretation \( I \) respects the role assertion \( r(a, b) \) iff \( (a^I, b^I) \in r^I \). \( I \) is a model of the ABox \( \mathcal{A} \) iff it respects all concept assertions and all role assertions from \( \mathcal{A} \).

We say that the individual \( a \) is an instance of the concept \( C \) with respect to \( \mathcal{A} \) iff \( a^I \in C^I \) holds for all models \( I \) of \( \mathcal{A} \).

Consider the ABox
\[
\mathcal{A} = \{ A(d), A(e), A(f), B(f), r(d, e), r(e, g), s(e, f), s(g, g), s(g, d) \}
\]

a) Present a graphical representation of the ABox.
b) For each of the \( \mathcal{ALC} \)-concepts \( C \) from Exercise 2, list all individuals that are instances of \( C \) w.r.t. \( \mathcal{A} \).
c) Compare your results to Exercise 2. Explain the differences.

Exercise 4
Consider the TBox
\[
\mathcal{T} = \{ \neg(A \sqcup B) \subseteq \bot, \ A \sqsubseteq \neg B \sqcap \exists r.B, \ D \sqsubseteq \forall r.A, \ B \sqsubseteq \neg A \sqcap \exists r.A \},
\]
the ABox
\[
\mathcal{A} = \{ r(a, b), \ r(a, c), \ r(a, d), \ r(d, c), \ (B \sqcap \forall r.D)(a), \ E(b), \ (\neg A)(c), \ (\exists s.\neg D)(d) \}
\]
and the ontology \( \mathcal{O} = (\mathcal{T}, \mathcal{A}) \). Check for
a) the TBox \( \mathcal{T} \)
b) the ABox \( \mathcal{A} \) and
c) the knowledge base \( \mathcal{O} \)
whether it has a model. If it has one, specify such a model. If it does not have a model, explain why.