TECHNISCHE UNIVERSITÄT DRESDEN

Faculty of Computer Science Institute of Theoretical Computer Science, Chair of Automata Theory

Fuzzy Description Logics

Exercise Sheet 1

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Exercise 1

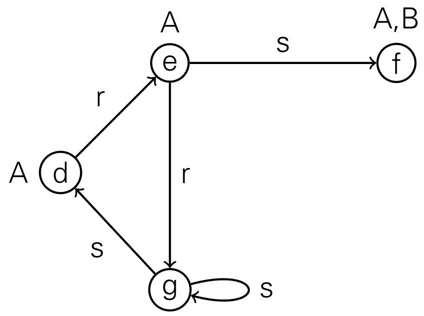
Recall that the description logic $\mathcal{ALC}$ is equipped with the concept constructors negation ($\neg$), conjunction ($\sqcap$), disjunction ($\sqcup$), existential restriction ($\exists r.C$), and universal restriction ($\forall r.C$). Each subset of this set of constructors gives rise to a fragment of $\mathcal{ALC}$.

Identify all minimal fragments that are equivalent to $\mathcal{ALC}$ in the sense that for every $\mathcal{ALC}$-concept, there is an equivalent concept in the fragment.

Two concepts are equivalent iff the concepts have the same extension in every interpretation.

Exercise 2

Consider the (graphical representation of the) interpretation $\mathcal{I}$ with $\Delta^{\mathcal{I}} = \{d, e, f, g\}$:



For each of the following $\mathcal{ALC}$-concepts C, list all elements x of $\Delta^{\mathcal{I}}$ such that $x \in C^{\mathcal{I}}$:

a) $A \sqcup B$

b) $\exists s.\neg A$

c) $\forall s.A$

d) $\exists s.\exists s.\exists s.\exists s.A$

e) $\neg\exists r.(\neg A \sqcap \neg B)$

f) $\exists s.(A \sqcap \forall s.\neg B) \sqcap \neg\forall r.\exists r.(A \sqcup \neg A)$

Exercise 3

In addition to the concept assertions presented in the lecture, ABoxes are sometimes allowed to contain role assertions of the form $r(a, b)$. An interpretation $\mathcal{I}$ respects the role assertion $r(a, b)$ iff $(a^{\mathcal{I}}, b^{\mathcal{I}}) \in r^{\mathcal{I}}$. $\mathcal{I}$ is a *model* of the ABox $\mathcal{A}$ iff it respects all concept assertions and all role assertions from $\mathcal{A}$.

We say that the individual a is an *instance* of the concept C with respect to $\mathcal{A}$ iff $a^{\mathcal{I}} \in C^{\mathcal{I}}$ holds for all models $\mathcal{I}$ of $\mathcal{A}$.

Consider the ABox

$$\mathcal{A} = \{A(d), A(e), A(f), B(f), r(d, e), r(e, g), s(e, f), s(g, g), s(g, d)\}$$

a) Present a graphical representation of the ABox.

b) For each of the $\mathcal{ALC}$-concepts C from Exercise 2, list all individuals that are instances of C w.r.t. $\mathcal{A}$.

c) Compare your results to Exercise 2. Explain the differences.

Exercise 4

Consider the TBox

$$\mathcal{T} = \{\neg(A \sqcup B) \sqsubseteq \bot, \quad A \sqsubseteq \neg B \sqcap \exists r.B, \quad D \sqsubseteq \forall r.A, \quad B \sqsubseteq \neg A \sqcap \exists r.A\},$$

the ABox

$$\mathcal{A} = \{r(a, b), \quad r(a, c), \quad r(a, d), \quad r(d, c), \quad (B \sqcap \forall r.D)(a), \quad E(b), \quad (\neg A)(c), \quad (\exists s.\neg D)(d)\}$$

and the ontology $\mathcal{O} = (\mathcal{T}, \mathcal{A})$. Check for

a) the TBox $\mathcal{T}$

b) the ABox $\mathcal{A}$ and

c) the knowledge base $\mathcal{O}$

whether it has a model. If it has one, specify such a model. If it does not have a model, explain why.