TECHNISCHE UNIVERSITÄT DRESDEN

Faculty of Computer Science Institute of Theoretical Computer Science, Chair of Automata Theory

Fuzzy Description Logics

Exercise Sheet 2

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Summer Semester 2013

Exercise 5

Can the following statements in natural language be expressed in $\mathcal{EL}$? If possible, give an $\mathcal{EL}$-axiom that captures their meaning.

- Superheroes that wear bat-costumes have sidekicks.
- Someone whose opponent is a superhero is a supervillain.
- Only superheroes and supervillains can have superpowers.

Exercise 6

Proof that existential restrictions are monotonic, i.e. show

$$C \sqsubseteq_{\mathcal{T}} D \;\rightarrow\; \exists r.C \sqsubseteq_{\mathcal{T}} \exists r.D.$$

Exercise 7

Consider the TBox $\mathcal{T}$ having the following axioms:

$$\begin{aligned} A &\sqsubseteq \exists r.(C \sqcap D), \\ B \sqcap \exists r.B &\sqsubseteq \exists r.\exists r.B, \\ \exists r.\exists r.A &\sqsubseteq B, \\ C &\sqsubseteq B \sqcap \exists r.A\}. \end{aligned}$$

Normalize $\mathcal{T}$ using the normalization rules

(NF1) $C \sqcap \hat{D} \sqsubseteq E \;\rightsquigarrow\; \hat{D} \sqsubseteq A, C \sqcap A \sqsubseteq E$,

(NF2) $\exists r.\hat{D} \sqsubseteq E \;\rightsquigarrow\; \hat{D} \sqsubseteq A, \exists r.A \sqsubseteq E$,

(NF3) $B \sqsubseteq \exists r.\hat{C} \;\rightsquigarrow\; A \sqsubseteq \hat{C}, B \sqsubseteq \exists r.A$,

(NF4) $\hat{C} \sqsubseteq \hat{D} \;\rightsquigarrow\; \hat{C} \sqsubseteq A, A \sqsubseteq \hat{D}$, and

(NF5) $C \sqsubseteq D \sqcap E \;\rightsquigarrow\; C \sqsubseteq D, C \sqsubseteq E$

where $\hat{C}, \hat{D} \notin \mathcal{N}_C \cup \{\top\}$ and A is a new concept name.

Exercise 8

Verify whether the subsumption relation

$$A \sqsubseteq \exists r.\exists r.B$$

holds with respect to the TBox $\mathcal{T}$ from Exercise 3 using the completion rules

(R1) $A_1 \sqcap A_2 \sqsubseteq B \in \mathcal{T}, A_1, A_2 \in S(A) \ \rightsquigarrow$ add B to $S(A)$,

(R2) $A_1 \sqsubseteq \exists r.B \in \mathcal{T}, A_1 \in S(A) \ \rightsquigarrow$ add r to $R(A, B)$, and

(R3) $\exists r.A_1 \sqsubseteq B \in \mathcal{T}, A_1 \in S(A_2), r \in R(A, A_2) \ \rightsquigarrow$ add B to $S(A)$,

where each concept name A is initially labelled with $S(A) = \{A, \top\}$ and each pair (A, B) is initially labelled with $R(A, B) = \emptyset$.

Exercise 9

Prove the following result.

Let $\mathcal{K} = (\mathcal{T}, \mathcal{A})$ be a knowledge base.

If a is an instance of C w.r.t. $\mathcal{K}$ and $C \sqsubseteq_{\mathcal{T}} D$, then a is an instance of D w.r.t. $\mathcal{K}$.