Fuzzy Description Logics

Exercise Sheet 4
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Exercise 15
Using ordinal sums, construct a continuous t-norm where exactly 3 values from $[0, 1]$ are idempotent, i.e. exactly 3 values satisfy $x \otimes x = x$.

Exercise 16
Which of the following t-norms (if any) are isomorphic?

2nd Hamacher t-norm: $x \otimes_H^2 y = \frac{xy}{xy-x-y+z}$

2nd Schweizer-Sklar t-norm: $x \otimes^{SS}_2 y = \sqrt{\max\{x^2 + y^2 - 1, 0\}}$

2nd Yager t-norm: $x \otimes_Y^2 y = \max\{1 - \sqrt{(1-x)^2 + (1-y)^2}, 0\}$

Exercise 17
Consider the fuzzy ABox

$$\mathcal{A} = \{(A(a) \geq 0.5), (r(a,b) \geq 0.9), (r(a,c) \geq 0.7)\}$$

and the fuzzy TBox

$$\mathcal{T} = \{(A \sqcup \forall r.(A \sqcup \neg B) \geq 0.8), (A \sqcup \exists r.\neg B \geq 0.9)\}$$

the Gödel t-norm is used. Present a non-crisp model of $\mathcal{A}$ and $\mathcal{T}$. 
Exercise 18

Consider a set of concept names

\[ \mathcal{N}_C = \{ \text{DifficultSubject}, \text{SmartPerson} \} \]

and a set of role names

\[ \mathcal{N}_R = \{ \text{likes}, \text{basedOn} \} \]

Let a fuzzy interpretation be given by the following graphical representation (absent edges are meant to be read as 0, e.g. in this example \( \text{likes}^F(\text{Student}, \text{Professor}) = 0 \)).

![Graphical representation]

For the Gödel t-norm, give the interpretations of the following concept descriptions.

a) \( \text{SmartPerson} \sqcap \exists \text{likes}. \text{DifficultSubject} \)

b) \( \forall \text{likes}. \exists \text{basedOn}. \text{DifficultSubject} \)

c) \( \exists \text{likes}. \forall \text{basedOn}. \text{DifficultSubject} \)