Fuzzy Description Logics

Exercise Sheet 5

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Exercise 21

Let $C$, $D$ be two $\mathcal{ALC}$-concept descriptions, $T$ a crisp $\mathcal{ALC}$-TBox and $a$ an individual name.

a) Show that $C$ is subsumed by $D$ wrt. $T$ iff $O = (T, \{(C \cap \neg D)(a)\})$ is inconsistent.

b) Give a Gödel-$\mathcal{ALC}$ TBox $T'$ (using only the relational operator $>$) and $\mathcal{ALC}$ concept descriptions $C'$ and $D'$ such that
   - $O' = (T', \{\langle(C' \cap \neg D')(a) > 0\rangle\})$ is inconsistent, and
   - $\deg(C' \sqsubseteq_{T'} D') = 0$.

Exercise 22

Let $\otimes$ be the product t-norm. Let $O = (T, A)$ be the $\otimes$-$\mathcal{EL}$ ontology consisting of the TBox $T = \{\langle \top \sqsubseteq A < 1 \rangle\}$ and the ABox $A = \{\langle \exists r. A(a) \geq 1 \rangle\}$.

a) Is $O$ consistent?

b) Does $O$ have a witnessed model?

Exercise 23

As an alternative to the Gödel-negation one can define the involutive negation $\sim$ whose semantics is defined as

$$(\sim C)^I(x) = 1 - C^I(x)$$

for all interpretations $I$ and all $x \in \Delta^I$. Using Gödel-$\mathcal{EL}$ and involutive negation construct a TBox $T$ and an ABox $A$ with the following properties.

a) There is a constant $q \in (0, 1)$ and a concept name $A$ occurring in $T$ such that $A^I(x) = q$ for all models $I$ of $T$ and all $x \in \Delta^I$.

b) $A$ is inconsistent but becomes consistent if all occurrences of $\neg$ are replaced by $\sim$. 
Exercise 24

Prove that the following equivalences hold for every t-norm $\otimes$ that is not nilpotent and its t-conorm and residuum.

a) $\mathbb{1}(x \oplus y) = \mathbb{1}(x) \oplus \mathbb{1}(y)$

b) $\mathbb{1}(x \Rightarrow y) = \mathbb{1}(x) \Rightarrow \mathbb{1}(y)$