TECHNISCHE UNIVERSITÄT DRESDEN

Faculty of Computer Science Institute of Theoretical Computer Science, Chair of Automata Theory

Fuzzy Description Logics

Exercise Sheet 5

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Exercise 21

Let C, D be two $\mathcal{ALC}$-concept descriptions, $\mathcal{T}$ a crisp $\mathcal{ALC}$-TBox and a an individual name.

a) Show that C is subsumed by D wrt. $\mathcal{T}$ iff $\mathcal{O} = \big(\mathcal{T}, \big\{(C \sqcap \neg D)(a)\big\}\big)$ is inconsistent.

b) Give a Gödel-$\mathcal{ALC}$ TBox $\mathcal{T}'$ (using only the relational operator $>$) and $\mathcal{ALC}$ concept descriptions C' and D' such that

- $\mathcal{O}' = \big(\mathcal{T}', \big\{\big\langle (C' \sqcap \neg D')(a) > 0 \big\rangle\big\}\big)$ is inconsistent, and
- $\mathrm{degree}(C' \sqsubseteq_{\mathcal{T}'} D') = 0$.

Exercise 22

Let $\otimes_\Pi$ be the product t-norm. Let $\mathcal{O} = (\mathcal{T}, \mathcal{A})$ be the $\otimes_\Pi$-$\mathcal{EL}$ ontology consisting of the TBox $\mathcal{T} = \{\langle \top \sqsubseteq A < 1 \rangle\}$ and the ABox $\mathcal{A} = \{\langle \exists r.A(a) \geq 1 \rangle\}$.

a) Is $\mathcal{O}$ consistent?

b) Does $\mathcal{O}$ have a witnessed model?

Exercise 23

As an alternative to the Gödel-negation one can define the *involutive negation* $\sim$ whose semantics is defined as

$$(\sim C)^{\mathcal{I}}(x) = 1 - C^{\mathcal{I}}(x)$$

for all interpretations $\mathcal{I}$ and all $x \in \Delta^{\mathcal{I}}$. Using Gödel-$\mathcal{EL}$ and involutive negation construct a TBox $\mathcal{T}$ and an ABox $\mathcal{A}$ with the following properties.

a) There is a constant $q \in (0, 1)$ and a concept name A occuring in $\mathcal{T}$ such that $A^{\mathcal{I}}(x) = q$ for all models $\mathcal{I}$ of $\mathcal{T}$ and all $x \in \Delta^{\mathcal{I}}$.

b) $\mathcal{A}$ is inconsistent but becomes consistent if all occurrences of $\neg$ are replaced by $\sim$.

Exercise 24

Prove that the following equivalences hold for every t-norm $\otimes$ that is not nilpotent and its t-conorm and residuum.

a) $\mathbb{1}(x \oplus y) = \mathbb{1}(x) \oplus \mathbb{1}(y)$

b) $\mathbb{1}(x \Rightarrow y) = \mathbb{1}(x) \Rightarrow \mathbb{1}(y)$